Comparison Shopping: Learning Before Buying From Duopolists

Brian C. Albrecht* Mark Whitmeyer†

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Abstract

We explore a model of duopolistic competition in which consumers flexibly learn about the fit–both relative and absolute–of each competitor's product. When information is cheap, increasing the cost of information decreases consumer welfare; but when information is expensive, this relationship flips: cheaper information hurts consumers. When the sellers' goods are both high and low value with positive probability, as information frictions vanish, the limiting equilibrium is efficient, in contrast to the monopoly model studied by Ravid et al. (2022).

*International Center for Law and Economics and Coles College of Business, Kennesaw State University. Email: mail@briancalbrecht.com

†Arizona State University. Email: mark.whitmeyer@gmail.com

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1 Introduction

Suppose your smartphone breaks and you need to buy a new one. A few aspects of this purchase stand out. First, there are multiple phones you could buy. Second, the technological differences between the phones are not perfectly clear, so the value to you of each phone is equally unclear. Third, there is a lot of information available online about the different phone options, but accessing or parsing it comes at a cost. Finally, the sellers of smartphones are not ignorant of these details; they set prices while taking into explicit account that they compete with other sellers and that consumers are learning about both products. Many markets, such as cars, appliances, electronics, and contractors, share these features.

We study a stylized model that incorporates flexible, costly learning by a buyer and competition between sellers. The model closely resembles Ravid, Roesler, and Szentes (2022) but with multiple sellers and horizontally differentiated products. Initially, the buyer only knows a prior distribution of the value of each object. However, the buyer can learn more about her value of each product at a cost. Like learning about smartphones on the internet, the consumer's ability to learn is extremely flexible: she can acquire *any* signal about her valuations. Signals are costly, but we assume this cost is smooth and strictly increasing in how informative the signal is, and the sellers set prices without learning the buyer's strategy or signal realization.

In the single seller case, Ravid, Roesler, and Szentes (2022) expose the troubling result that in a bilateral-trade setting with "learning before trading," as information costs go to zero, the limit equilibrium is *extremely* inefficient. To be more precise, they show there exists a continuum of equilibria when information is free. As information costs vanish, the equilibrium converges to the *worst* free-learning equilibrium. Even though trade is always beneficial and information is free, the consumer does not purchase with strictly positive probability. Ours is the oligopolistic analog of Ravid, Roesler, and Szentes (2022): we wish to understand whether and how competition potentially alters this result. In their paper, a hold-up problem generates inefficiencies. Does the competition of our setting mitigate this issue?

Our main result proves that, with multiple sellers, as the information costs vanish, the limiting equilibrium is efficient. If the main takeaway from Ravid, Roesler, and Szentes (2022) is "that possessing information might be significantly better than having cheap access to it," one takeaway from our paper is that competition overturns that result: possessing cheap information is similar to having cheap access to it. Along the way, we must solve a pricing game between the firms, conditional on the level of learning attained. As in other papers, the equilibrium pricing strategies involve a distribution of prices that generate unit-elastic residual demand curves (Albrecht, 2020; Condorelli and Szentes, 2020).

The more difficult part of constructing the equilibrium is solving for optimal learning, given the pricing strategies. A learning strategy means picking a distribution of posteriors that is Bayes' plausible. First, we prove that it is always optimal for the buyer to choose learning strategies that reveal only the relative value. We call this "comparison shopping." Vaguely, if (x, y) is the consumer's valuation for goods from seller one and seller two, she learns only along some line $y = A - x$. For example, it is optimal to learn sometimes that good one is λ better than good two and sometimes learn good two is λ better. As information becomes cheaper, the consumer acquires posteriors further and further from the prior. One complication that arises in constructing the optimal learning is that they eventually "run into the boundary" of the square. In that situation, they "leave" beliefs at the prior.

The equilibrium requires solving a multidimensional information design problem on top of an equilibrium pricing game. In our problem, the value of posterior beliefs is not exogenous but an endogenous object that depends on the firms' pricing, which is random. We can prove that in equilibrium, the consumer learns the relative value, i.e., *comparison shops*, but that is endogenous, not exogenous. To make progress, we impose various assumptions. For example, the boundary conditions mentioned create problems for our equilibrium construction, meaning we cannot always solve the problem for a general distribution of beliefs. Instead, much of the paper focuses on the case when each firm's product takes on one of two values.[1] Reassuringly, we show when learning

[1] Ravid et al. (2022) assume that the consumer's valuation is distributed on the unit interval according

costs are sufficiently high, there is an analogous equilibrium for a distribution of beliefs so that this binary-value assumption is innocuous.

After constructing the equilibrium, we conduct simple comparative statics to ask questions like "when does cheaper information increase consumer welfare?" Even without the cost of acquiring information, there is a trade-off for the consumer.[2] If she acquires more information, she has a better match quality with the seller, but that softens competition between the sellers. If she remains ignorant, she can induce stronger price competition, but her lack of information hurts her purchase decision. When the cost of information is high, consumer welfare is decreasing in the cost of information, but when information is cheap, this relationship flips. In Section 6 we show that these comparative statics persist beyond the binary-match-value environment.

The rest of the paper is as follows: Section 1.1 covers related work, Section 2 sets up the model, Section 3 sets up the benchmark where sellers observe what the buyers learn, Section 4 solves the optimal pricing game, given the information acquired, Section 5 solves for optimal learning, given the equilibrium pricing, and proves the main results, and finally Section 6 illustrates how much of our analysis carries over to the case when the consumer's prior has a density.

1.1 Related Work

Beyond the closest paper to ours, Ravid et al. (2022), there is now a sizeable collection of papers that study the important question, *"what can happen in markets under different information structures?"* Bergemann et al. (2021) asks this in the context of a search market, and Bergemann et al. (2015) characterizes possible market outcomes when there is a single monopolistic seller. Condorelli and Szentes (2022) look at Cournot competition through this lens, while Roesler and Szentes (2017) and Condorelli and Szentes (2020) study consumer-optimal information and distributions over valuations, respec-

to some prior with a density. Their inefficiency result persists if the consumer's valuation is either high or low, instead.

[2] Many papers explore this trade-off, e.g., Moscarini and Ottaviani (2001), Armstrong and Zhou (2016), and Albrecht (2020).

tively. Armstrong and Zhou (2022), Albrecht (2020), Shi and Zhang (2022), Elliott et al. (2019), Rhodes and Zhou (2022), Armstrong and Vickers (2022), and Dogan and Hu (2022) all study variations on this theme in markets with imperfect competition. Of special note is Moscarini and Ottaviani (2001), who study price competition by duopolists who face a privately informed buyer. Crucially, in their work the consumer's information is exogenous, and they study the pricing-only game between the firms. Our work incorporates explicit information acquisition, but since we allow any signal, it is similar in spirit to the papers above.

There are a number of papers that explore information acquisition in markets by consumers. Many limit their analysis to the monopolist scenario. Branco et al. (2012), Branco et al. (2016), Pease (2018), and Lang (2019) all look at how a monopolist sells to consumers who may subsequently acquire information about their valuation for the product. Importantly, in these papers, the consumer observes the firm's price before deciding how and what to learn. This timing is also assumed in the oligopolistic setting of Matějka and McKay (2012).[3] Jain and Whitmeyer (2023) explore flexible information acquisition by consumers in a large oligopolistic market with search frictions á la Wolinsky (1986). Like this paper, the primary focus there is the case in which the consumer acquires information before observing a firm's price offer.

Finally, as we stated in the introduction, our equilibrium construction requires solving a multidimensional information design problem along with a pricing equilibrium. We use recent technical developments in multidimensional Bayesian persuasion and information design settings. More specifically, we use results and insights from Dworczak and Kolotilin (2019), Yoder (2021), and Kleiner et al. (2023).

[3]The literature on information acquisition in auctions/mechanism design (e.g., Persico (2000), Shi (2012), Kim and Koh (2022), Mensch (2022), and Thereze (2022)) also imposes that the mechanism is publicized–and committed to by the designer–before consumers acquire information.

2 Model

There are 2 *ex ante* identical horizontally differentiated firms, indexed by i. Each of them is selling a product whose value to a representative consumer is a identically distributed binary random variable Z_i with support on $\{0,1\}$ and $\mu := \mathbb{P}(Z_i = 1) \in (0,1)$ for all i. We also assume that the joint distribution of the random variables is symmetric: $\mathbb{P}(Z_1 = 1 | Z_2 = v) = \mathbb{P}(Z_2 = 1 | Z_1 = v)$ for all $v \in \{0,1\}$. We denote $\omega := \mathbb{P}(Z_2 = 1, Z_1 = 0)$.

For many of the results we specialize to the following three cases:

(i) **Positive correlation not too high, and prior not too extreme:** μ and ω are such that

$$0 < \omega \leq \frac{4}{5} \min\{\mu, 1-\mu\}.$$

Note that this specializes to $\frac{1}{5} \leq \mu \leq \frac{4}{5}$ in the i.i.d. case.

(ii) **At most one product is high value:** $\mathbb{P}(Z_1 = 1, Z_2 = 1) = 0$, i.e., $\omega = \mu$.

(iii) **At most one product is low value:** $\mathbb{P}(Z_1 = 1, Z_2 = 0) = 0$, i.e., $\omega = 1 - \mu$.

The consumer privately learns about the state of the world at a cost, formalized as follows. Let $\mathcal{F}$ denote the set of all distributions supported on $[0,1]^2$ that are fusions of the prior, i.e., that can be obtained by observing some signal. The consumer may acquire any fusion $F \in \mathcal{F}$ at cost $C\colon \mathcal{F} \to \mathbb{R}$, where C satisfies the following assumptions:

Assumptions on the Cost Functional: We assume for any $F \in \mathcal{F}$,

$$C(F) = \kappa \int c dF\ ,$$

for some strictly convex, thrice differentiable, function c and scalar $\kappa > 0$, with $c(\mu,\mu) = 0$, $c(x,y) < \infty$ for all $(x,y) \in (0,1)^2$, and $\lim_{(x',y') \to (x,y)} \left|Dc(x',y')\right| = \infty$ for all $(x,y) \in \partial[0,1]^2$.

We assume further that c is symmetric: for any permutation $\sigma(x,y)$ of vector (x,y), $c(x,y) = c(\sigma(x,y))$. We also make the following technical assumption: the third directional derivative of c in the direction of $(1,-1)$ is negative: $c_{xxx} - c_{yyy} + 3c_{xyy} - 3_{xxy} \leq 0$.

Note that our symmetry assumption is obviously satisfied by any cost function of the form

$$c(x,y) = d(x) + d(y) \ ,$$

where $d\colon [0,1] \to \mathbb{R} \cup \infty$ is some strictly convex function. For any figures, we will use the following function:

$$\begin{aligned} c(x,y) =& x \log x + (1-x)\log(1-x) + y \log y + (1-y)\log(1-y) \\ & - 2(\mu \log \mu + (1-\mu)\log(1-\mu)) \end{aligned} . \tag{☿}$$

We also specify that the consumer's utility is additively separable in her value for the good she purchases, its price, and her cost of acquiring information: if she purchases a product with expected value x at price p and at posterior (x,y), her utility is $x - p - c(x,y)$. We assume that the consumer has a negligible (or nonexistent) outside option, and so she will always purchase from one of the firms. For simplicity, we normalize the marginal costs of production for the two firms to 0.

The timing of the game is straightforward:

(i) **Private Learning:** The consumer acquires information about the two products. Neither firm observes this learning.

(ii) **Simultaneous Price Setting:** The firms simultaneously post prices.

(iii) **Purchase Decision** Given posterior value (x,y) and prices (p_1,p_2), the consumer purchases from Firm 1 (2) if $x - p_1 > \ (<) \ y - p_2$ and breaks ties fairly if she is indifferent.

In the first (information acquisition) stage, the consumer solves

$$\max_{F \in \mathcal{F}} \int (u - c)\, dF,$$

where $u\colon [0,1]^2 \to \mathbb{R}$ is the consumer's reduced form utility from posterior (x,y).

2.1 Discussion of the Setup

Let us briefly discuss, explain a few of our assumptions.

Binary Values: We assume that each firm's product takes just one of two values. When frictions are large, this is inconsequential: we show in Section 6 that unless κ is too small, there is an analogous equilibrium to the one we construct in our main specification when the consumer's value for the two firms' products is distributed according to some density on $[0,1]^2$. It is when information is cheap that we run into trouble: the learning with three-point support that we identify is no longer optimal for the consumer. Instead, we conjecture that the consumer now acquires a continuum of posteriors close to the prior plus possible point masses on more extreme posteriors. This is merely a conjecture; however, finding an equilibrium in the firm's pricing game has proved to be beyond our abilities.

Symmetric Firms: Like the previous assumption, this is for tractability. The equilibrium of the pricing-only game becomes quite difficult to construct when firms are asymmetric.

Parametric Assumption on the Prior: We make a rather cryptic stipulation that the positive correlation between the consumer's values for the two firms' goods cannot be too high nor can the prior be too extreme. This is again due to the challenges in constructing an equilibrium in the pricing game between the firms: it is much easier to construct an equilibrium in the pricing-only game when the probability of the "tie" belief (when the consumer's distribution has symmetric three-point support) is sufficiently high. Our parametric assumption, thus, guarantees this is true in the consumer's optimal learning when frictions are sufficiently low.

Private Learning Before Trading: We assume that the consumer learns before she observes the firms' prices and that; moreover, this learning is private.[4] In the next section, we allow for public learning and show that a hold-up problem emerges, which leads to zero information acquisition by the consumer. In addition, the timing in our main environment (learning before trading) is realistic in many environments: in particular, learning about a service provider's reputation seems especially fitting. Our timing assumption is also that made by Ravid et al. (2022), which allows us to cleanly identify the effects of competition.

[4] Matějka and McKay (2012) study a related scenario in which firms set prices before the consumer learns.

Cost Function: The consumer's cost of acquiring information is a linear functional of the distribution over posteriors. We assume this posterior-separable form for tractability. Moreover, we specify that the convex function that is integrated has unbounded slope at the boundaries of the unit square. This is done to ensure an "interior" solution in the consumer's information acquisition problem. Importantly, this specification only makes our convergence result more difficult to attain: if the slope were bounded our results would go through with the modification that the results would no longer be limit results but would hold for sufficiently small positive κ.

3 Observable Learning Benchmark

A natural benchmark is the case in which the firms observe the consumer's acquired posterior $(x, y) \in [0,1]^2$ before posting prices. The first step in characterizing the equilibrium is to characterize equilibria in the pricing-only game between the two firms for an arbitrary vector (x, y).

We define the distributions Υ_1 and Υ_2 as

$$\Upsilon_1(p) := 1 - \frac{\underline{p} + \lambda}{p + \lambda}, \quad \text{on} \quad \left[\underline{p}, \infty\right),$$

and

$$\Upsilon_2(p) := 1 - \frac{\underline{p}}{p - \lambda}, \quad \text{on} \quad \left[\underline{p} + \lambda, \infty\right),$$

where $\underline{p} \in \mathbb{R}_{++}$ and $\lambda := y - x > 0$.

Lemma 3.1. *For all (x, y), an equilibrium of the pricing-only game exists. If $x < y$, there exist a continuum of equilibria, parametrized by $\underline{p}$, in which firm 1 chooses distribution Υ_1 and firm 2 chooses Υ_2. For any (x, y) with $x = y$, the unique equilibrium is the Bertrand outcome: both firms price at marginal cost, $p_1 = p_2 = 0$.*

It is straightforward to check that the equilibria constructed in Lemma 3.1 are particularly bad for the consumer: her expected payoff at any (x, y) with $y \neq x$ is strictly negative. Moreover, it is clear that the consumer's net payoff at any (x, y), in any equilibrium, must be as follows:

Lemma 3.2. *In any equilibrium of the pricing-only game with $y > x$, the expected net payoff to the consumer is weakly less than x.*

Working backward, we now conclude that the consumer will not learn.

Proposition 3.3. *If $\kappa > 0$, the unique equilibrium with observable learning is for the consumer to acquire no information: she chooses the degenerate distribution on the prior (μ, μ).*

4 Two Pricing Games Between Firms

With a view toward characterizing the symmetric equilibria of the game with flexible learning by the consumer and price-setting by the firms, we start by examining two games of pure price setting, holding fixed the consumer's learning; *viz.*, fixing some exogenous distribution over consumer valuations for the two firms' products. In the first pricing game, the distribution over valuations has symmetric support where the valuation for one product is favored by some amount $\lambda > 0$. In the second pricing game, there is also support at the prior, where the products are valued equally. In Section 5, we then show these are the equilibrium valuations after the consumer acquires information in equilibrium.

4.1 Value Distribution With Symmetric n Point Support

For this subsection, we can easily generalize from two to n firms.[5] Suppose the consumer has a symmetric distribution over valuations for the firms' products with support on n points as follows. At each of the n points, her valuation for $n-1$ of the firms is the same, and her valuation for one of the firms is $\lambda > 0$ greater than that of the others. That is, after normalization, with probability $\frac{1}{n}$, the consumer's vector of valuations for the n firms is $(0, \ldots, 0, \lambda, 0, \ldots, 0)$. The consumer prefers one firm λ more than all the others, which she considers equal. This game is closely related to that of Moscarini and Ottaviani (2001)

[5] This is the only part of the paper in which we study a general oligopolistic environment, rather than merely duopoly. Our reason for including this generalization is that we think that the limit result of Proposition 4.3 is interesting.

with two modifications: first, the consumer's outside option is negligible; and second, there are $n \geq 2$ firms, rather than just two. We also impose symmetry, which we justify later, when we endogenize the consumer's learning.

Lemma 4.1. *There exist no symmetric equilibria in which firms do not randomize over prices. Moreover, firms' distributions over prices cannot have atoms.*

Proof. If such an equilibrium exists, a firm's demand is (locally) perfectly inelastic–if it raises its price slightly, the consumer will purchase from it with the same probability, yielding strictly higher profits. ■

As a result, we search for an equilibrium in which firms randomize over prices. Define the distribution $\Gamma(p)$ as

$$\Gamma(p) := \begin{cases} \Gamma_L(p), & \beta\lambda \leq p \leq \lambda(1+\beta) \\ \Gamma_H(p), & \lambda(1+\beta) \leq p \leq \lambda(2+\beta) \end{cases}, \tag{♀}$$

where

$$\Gamma_L(p) := 1 - \left(\frac{\beta(1+\lambda)}{p+\lambda}\right)^{\left(\frac{1}{n-1}\right)}, \quad \text{and} \quad \Gamma_H(p) := 1 - \frac{\lambda(2+\beta)-p}{(p-\lambda)(n-1)}\left(\frac{p}{\lambda(1+\beta)}\right)^{\left(\frac{n-2}{n-1}\right)},$$

and where β is the unique solution to

$$I(\beta) := \left(\frac{1+\beta}{2+\beta}\right)^{\left(\frac{1}{n-1}\right)} - \frac{1}{\beta(n-1)} = 0\,.$$

Proposition 4.2. *In the price-setting game of this subsection, the unique symmetric equilibrium is for each firm to choose the distribution over prices Γ specified in Expression ♀.*

This equilibrium distribution (in Expression ♀) arises as the distribution that generates unit-elastic demand for the other firm (given the exogenous uncertainty), rendering it, in turn, willing to randomize. One easy case is that in which there are just two firms. There,

$$\Gamma(p) = \begin{cases} \Gamma_L := \frac{p-\sqrt{2}\lambda}{\lambda+p}, & \sqrt{2}\lambda \leq p \leq \left(1+\sqrt{2}\right)\lambda \\ \Gamma_H := \frac{\left(3+\sqrt{2}\right)\lambda-2p}{\lambda-p}, & \left(1+\sqrt{2}\right)\lambda \leq p \leq \left(2+\sqrt{2}\right)\lambda \end{cases}, \tag{♁}$$

since, in this case, $\beta = \sqrt{2}$.

We can now ask what happens as the number of firms increases:

Proposition 4.3. *As the number of firms, n, increases, β (and so the lower bound for the distribution over prices) decreases strictly. As $n \uparrow \infty$, $\beta \downarrow 0$ and the limiting equilibrium distribution is the degenerate mass point on λ.*

In the limit, there is a form of monopolistic competition. Each firm sets a price equal to λ, the difference between a consumer's valuation for it (when it is the preferred firm) and the consumer's "outside option," 0. The reason is how new firms are added to the space of valuations. As in Perloff and Salop (1985), all firms are competitors with each other, and each new product is a new dimension but not necessarily a closer substitute than the previous product. That is different from a standard Hotelling, spatial competition model.

4.2 Value Distribution With Symmetric $n+1$ Point Support

Now suppose there are just two firms and the consumer has a symmetric distribution over valuations for the firms' products with support on 3 points as follows. For 2 points, her valuation for one of the firms is $\lambda > 0$ greater than that of the other firm. At the 3rd point, the consumer is indifferent between each of the firms.

After normalization, we specify that with probability $q \leq .405$ the consumer's vector of valuations for the 2 firms is $(0, \lambda)$ and with probability $1-2q$ the consumer's vector of valuations is $(0,0)$.

Lemma 4.4. *There exist no symmetric equilibria in which firms do not randomize over prices. Moreover, firms' distributions over prices cannot have atoms.*

Proof. Because the consumer is indifferent between the two firms with strictly positive probability, a standard under-cutting argument eliminates any symmetric equilibria in which a firm sets some price with strictly positive probability. ∎

Again, we search for an equilibrium in which firms randomize over prices. Define the distribution $\Phi(p)$

$$\Phi(p) := \frac{(1-q)(p(1-2q) - \lambda q)}{p(1-2q)^2} \quad \text{on} \quad \left[\frac{q}{1-2q}\lambda, \frac{q}{1-2q}\lambda + \lambda\right]. \tag{♂}$$

Proposition 4.5. *In the price-setting game of this subsection, it is an equilibrium for each firm to choose the distribution over prices Φ specified in Expression ♂.*

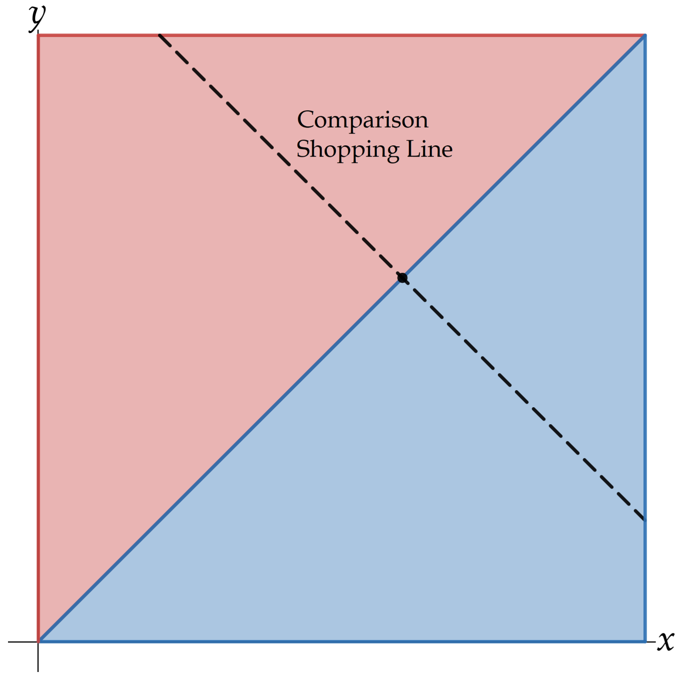


Figure 1: The Space of Valuations

5 Consumer Learning

Now, we will take as given the price-setting by the firms and search for equilibria in the grand game. We begin with the following theorem, which argues that as long as there exist frictions, no matter how small ($\kappa > 0$), the consumer only learns along the **Comparison Shopping** line $y = 2\mu - x$. That is, the consumer's learning exclusively focuses on the *relative* merits of each firm's product. Defining the set ℓ^* as

$$\ell^* := \left\{(x, y) \in [0,1]^2 \ : \ y = 2\mu - x\right\},$$

and saying that the consumer **Comparison Shops** if her acquired distribution over posteriors is supported on a subset of ℓ^*, our formal result is

Theorem 5.1. *If firms choose symmetric, atomless, distributions that admit densities with support on some closed interval* $\left[\underline{p}, \bar{p}\right]$*, the consumer comparison shops.*

The crucial observation behind this theorem is that the consumer's payoff as a function of her posterior is strictly concave along the vector orthogonal to the vector $(-1, 1)$. Bayes' plausibility (or a variant thereof) then pins down the comparison shopping line $2\mu - x$.

Figure 1 illustrates the space of valuations and the comparison shopping line. Posteriors in the red (blue) region are those at which the consumer's valuation for firm 2's (1's) good is highest. The dotted diagonal line is the comparison shopping line when the prior is the specified point.

5.1 Solving the Information Acquisition Problem

In solving the consumer's problem, we conjecture its solution and use the corresponding strategies by the firms in the pricing-only game to generate the consumer's value function. Then, we verify that the consumer's optimal learning is precisely that that we conjectured.

The value function is

$$\begin{aligned} V(x,y) :=& \mathbb{P}(x-p_1 \geq y-p_2)\,\mathbb{E}(x-p_1|x-p_1 \geq y-p_2) \\ &+ \mathbb{P}(y-p_2 \geq x-p_1)\,\mathbb{E}(y-p_2|y-p_2 \geq x-p_1) - \kappa c(x,y), \end{aligned}$$

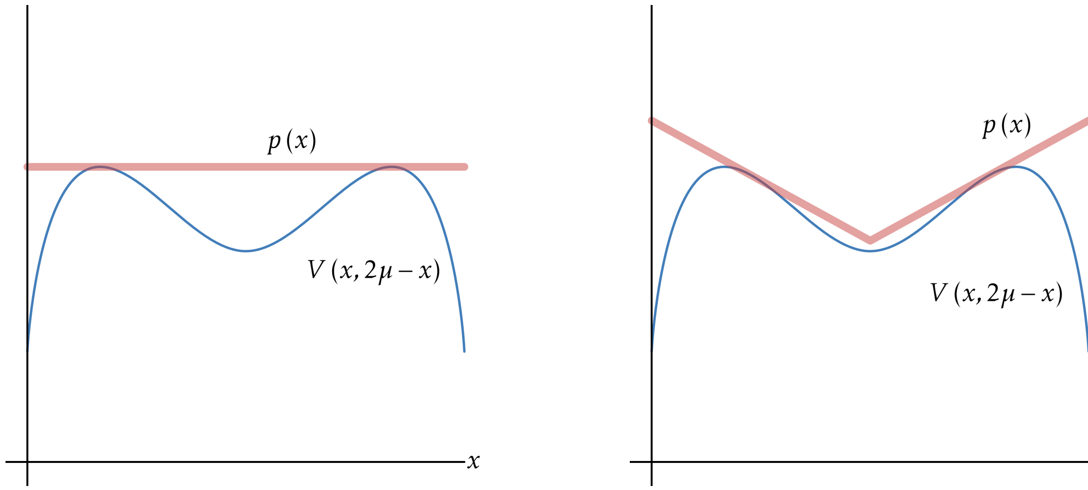
which is continuously differentiable except on $\partial[0,1]^2$ and is bounded above on the entire square. Accordingly, by Theorem 1 of Dworczak and Kolotilin (2019), we have weak duality and the price function solution lies weakly above the consumer's value in her information acquisition problem. From there, it is easy to solve the dual problem and verify that this corresponds to a solution to the primal problem. Alternatively, we can make use of the symmetry of the information acquisition problem and "split" the prior probabilities of $(1,1)$ and $(0,0)$ equally between the triangles

$$\Delta^1 := \left\{(x,y) \in [0,1]^2 : 0 \leq x \leq 1 \;\&\; 0 \leq y \leq 1-x\right\}, \tag{4}$$

and

$$\Delta^2 := \left\{(x,y) \in [0,1]^2 : 0 \leq x \leq 1 \;\&\; 1 \geq y \geq 1-x\right\}; \tag{5}$$

before then solving two standard 3-state persuasion problems (as any simplex is homeomorphic to the standard simplex), on each of the two triangles. These problems satisfy the assumptions of Yoder (2021), whose Proposition 2 reveals that the concavification approach is valid. It remains to verify that the maximum of the two concavifying planes is convex and lie every above the value function, and that the two planes either are the same or have $y=x$ as their intersection.



(a) Expensive Information: $\kappa \geq \bar{\kappa}$ (b) Moderately Costly Information: $\kappa \in [\underline{\kappa}, \bar{\kappa}]$

Figure 2: The two possibilities when information is not cheap (or at most one product is high, or low, value) with the ☿ cost.

The solution is then as follows. If information is expensive, if at most one product is high value, or if at most one product is low value, the price function is just a single plane; i.e., the two concavifying planes are the same plane. If information is moderately expensive (and both products can be high value) the price function is the maximum of two planes that intersect at $y = x$ and lie weakly above the value function on that line. Finally, if information is cheap, the price function is as in the moderate cost case, with the additional specification that it is equal to the value function at its minimum, along the line $y = x$.

5.2 Equilibrium for Expensive Information or When At Most One Product is High/Low Value

The easiest scenario to analyze is that in which either information is expensive (κ is large), the consumer's values for the two firms' products cannot both be 1, or the consumer's values for the two products cannot both be 0. Our main result of this section is that if frictions are sufficiently large, then regardless of the prior, there is an equilibrium in which the consumer acquires a binary distribution over posteriors.

In addition, if at most one product has high value or at most one product has low value (which includes as a special case the perfect negative correlation scenario), there is such an equilibrium regardless of the size of the friction. Given this we can characterize the limit result as information frictions vanish. As κ increases, the consumer's binary distribution moves out from (μ, μ) symmetrically along the comparison shopping line. In the limit, these two points hit the boundary of the unit square: the distribution converges to a binary distribution with support on $(\max\{2\mu-1, 0\}, \min\{1, 2\mu\})$ and $(\min\{1, 2\mu\}, \max\{2\mu-1, 0\})$. This equilibrium is not efficient: with positive probability the consumer purchases from a firm whose value (0) is strictly lower than that of the other firm (1).

This inefficiency result is a consequence of the equilibrium pricing strategies chosen by the two firms when the consumer's distribution over posteriors has symmetric binary support. Note that in the frictionless limit, the consumer either knows with certainty that one of the firms has high value (when $\mu \geq \frac{1}{2}$) or that one of the firms has low value (when $\mu \leq \frac{1}{2}$). Nevertheless, she still may purchase from the low-(or possibly low)-value firm as it's price may be significantly lower.

We say that a consumer **Comparison Shops With Uniform 2-Point Support** if the consumer's acquired distribution over valuations is supported on

$$\left\{\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right), \left(\mu + \frac{\lambda}{2}, \mu - \frac{\lambda}{2}\right)\right\},$$

each with probability $\frac{1}{2}$.

Theorem 5.2. *If κ is sufficiently high, if at most one product has high value, or if at most one product has low value, there is an equilibrium in which the consumer comparison shops with uniform 2-point support and firms randomize over prices according to Expression 6.*

As frictions decrease (κ dwindles) λ grows. Then,

Corollary 5.3. *If at most one product has high value or if at most one product has low value, then as information costs vanish ($\kappa \downarrow 0$) the limiting equilibrium is not efficient.*

Notably, when κ is sufficiently large, λ is strictly increasing in κ: as frictions shrink, the consumer learns more and more in a mean-preserving spread sense. For all such κ,

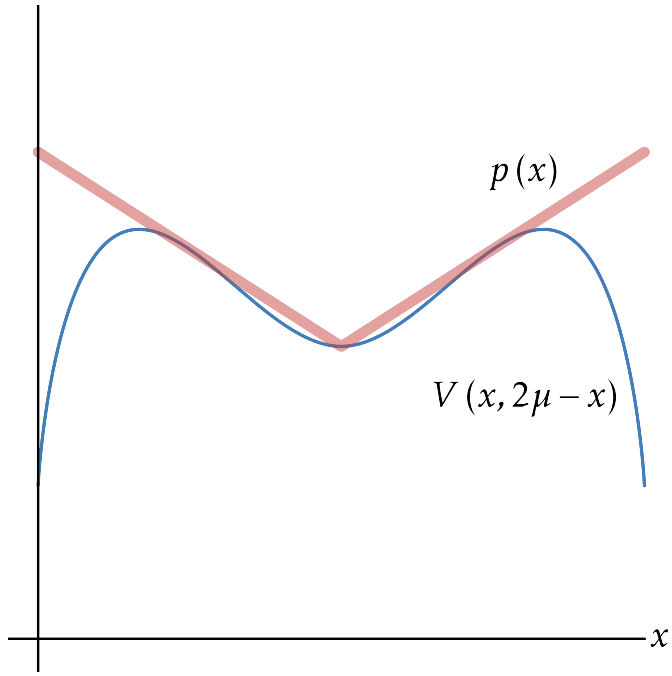


Figure 3: Cheap Information Equilibrium

the price function is a single plane with zero slope. Eventually (as κ continues to shrink), unless at most one product has high value, κ hits a threshold $\bar{\kappa}$. Then, for all κ within some interval $[\underline{\kappa}, \bar{\kappa}]$ the consumer's learning is the same–as are the pricing strategies by the firms. Here, the price function is the maximum of two planes whose intersection is the line $y = x$. Both cases are depicted in Figure 2, where we have substituted in $y = 2\mu - x$ (thanks to Theorem 5.1).

As neither the firms' behavior nor the consumer's learning are changing, but information is becoming cheaper, the consumer's welfare is strictly decreasing in κ on this interval. On the flip side, when $\kappa \geq \bar{\kappa}$ the opposite relationship exists. Summing things up:

Proposition 5.4. *For intermediate information costs, ($\kappa \in [\underline{\kappa}, \bar{\kappa}]$), the consumer's welfare is strictly decreasing in the size of the friction. For large information costs, ($\kappa \geq \bar{\kappa}$), the consumer's welfare is strictly increasing in the size of the friction.*

5.3 Equilibrium for Cheap Information

We say that a consumer **Comparison Shops With Occasional Indifference** if her acquired distribution over valuations has support on 3 points $\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right)$, (μ, μ) and $\left(\mu + \frac{\lambda}{2}, \mu - \frac{\lambda}{2}\right)$.

Theorem 5.5. *If κ is sufficiently low there is an equilibrium in which the consumer comparison shops with occasional indifference and firms randomize over prices according to Expression ♂.*

Corollary 5.6. *As information costs vanish $\kappa \downarrow 0$ the limiting equilibrium is efficient.*

The limiting equilibrium has support on three points

$$\{(\max\{2\mu - 1, 0\}, \min\{1, 2\mu\}), (\min\{1, 2\mu\}, \max\{2\mu - 1, 0\}), (\mu, \mu)\}.$$

In contrast to the previous subsection, in the limiting equilibrium, the firms price so that the consumer purchases from the advantaged firm–if there is an advantaged firm after her learning–with probability 1. The consumer never purchases from a firm that is worse than the other *ex post*. The consumer's learning in this equilibrium is depicted in Figure 3.

6 Extension to a Prior with a Density

An exact analog of Theorem 5.2 holds when the consumer's valuations for the two products are symmetrically distributed with nonzero density h on the unit square and the consumer's utility is affine in her valuation for the purchased product and additively separable in her valuation, the price, and the cost of acquiring information (which is posterior-mean measurable).

That is, suppose each firm is selling a product whose value to the consumer is a random variable Z_i with full support on $[0,1]$. Random vector (Z_1, Z_2) is distributed on $[0,1]^2$ according to continuous density $h(z_1, z_2)$, which is symmetric around the diagonal $y = x$, i.e., $h(z_1, z_2) = h(z_2, z_1)$ for all $z_1, z_2 \in [0,1]$. $\mu := \int_0^1 \int_0^1 af(a,b)\,db\,da$ denotes the prior expected value.

The consumer may acquire any fusion $G \in \mathcal{F}_H$ of the prior at cost $C(G) = \kappa \int c\,dG$ where we maintain the assumptions from the model section above. Then,

Proposition 6.1. *If κ is sufficiently high, there is an equilibrium in which the consumer comparison shops with uniform 2-point support and firms randomize over prices according to Expression ♁.*

Proof. This result is an immediate implication of the fact that in the proof of Theorem 5.2, we show that as κ increases, λ decreases and in the limit goes to 0. For any prior as specified in this section, there is a threshold $\lambda > 0$ such that the comparison shopping with uniform 2-point support distribution is a fusion of the prior. That we can use the same price-function approach follows from Dworczak and Kolotilin (2019). Alternatively, as Kleiner et al. (2023) establish, these distributions are exposed (in their parlance, "strongly exposed") points in the set of finitely-supported fusions of the prior. When frictions are high ($\kappa \geq \bar{\kappa}$), the associated power diagram is the trivial power diagram consisting of a single element ($[0,1]^2$). When frictions are moderate ($\kappa \in [\underline{\kappa}, \bar{\kappa}]$), the associated power diagram is convex partitional, with two elements, the triangles Δ^1 and Δ^2 (Expressions ♃ and ♄). ■

The comparative statics from Proposition 5.4 also carry over:

Proposition 6.2. *For intermediate information costs, ($\kappa \in [\underline{\kappa}, \bar{\kappa}]$), the consumer's welfare is strictly decreasing in the size of the friction. For large information costs, ($\kappa \geq \bar{\kappa}$), the consumer's welfare is strictly increasing in the size of the friction.*

The intuition is also the same: in the intermediate-friction region, the consumer's optimal learning; and, therefore, the firms' behavior, stays the same as κ dwindles. The consumer accrues all of the benefits of cheaper information. When κ is large, the consumer's learning is affected and so firms raise their prices (on average) to take advantage of their greater market power. This (negative, for the consumer) force is dominant, and so cheaper information makes the consumer worse off.

7 Conclusion

This paper develops a model of flexible information acquisition with imperfect competition between sellers. The buyer can purchase any signal about her valuation privately, but signals are costly. Without observing the buyer's learning strategy or outcome, sellers set prices and compete for the buyer.

Technical difficulties arise because the equilibrium requires solving a multidimensional information design problem on top of an equilibrium pricing game that involves a distribution of prices. The value of posterior beliefs is endogenous and depends on the firms' pricing, which is random. We prove that the consumer only wants to learn the relative values, which we call comparison shopping.

Our main result proves that competition between sellers flips the inefficiency result of Ravid, Roesler, and Szentes (2022). With multiple sellers, as the cost of information vanishes, the equilibrium outcome is efficient: the consumer always purchases the higher-value product. We also do comparative statics: when the cost of information is high, consumer welfare decreases in the cost of information, but when information is cheap, this relationship flips.

To progress on a difficult multidimensional information design problem, we impose various assumptions, such as whether the value is either high or low. We show that this assumption is sometimes unimportant. When learning costs are sufficiently high, there is an analogous equilibrium for a prior with a density so that this binary-value specification is innocuous. However, when information costs are low, our equilibrium construction does not carry over. Such constructions are left for future work.

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A Omitted Proofs and Derivations

A.1 Lemma 3.1 Proof

Recall that we want to show that the family of distributions

$$\Upsilon_1(p) := 1 - \frac{\underline{p} + \lambda}{p + \lambda}, \quad \text{on} \quad \left[\underline{p}, \infty\right),$$

and

$$\Upsilon_2(p) := 1 - \frac{\underline{p}}{p - \lambda}, \quad \text{on} \quad \left[\underline{p} + \lambda, \infty\right),$$

where $\underline{p} \in \mathbb{R}_{++}$ and $\lambda := y - x > 0$ constitute an equilibrium, for any fixed vector (x, y) (with $0 \leq x < y \leq 1$).

Proof. The symmetric case is immediate. In the asymmetric case, firm 1's profit function, given an equilibrium strategy by firm 2 is

$$\Pi(p) = \begin{cases} p, & \text{if} \quad 0 \leq p \leq \underline{p} \\ p\left[1 - \Upsilon_2(p + \lambda)\right] = \underline{p}, & \text{if} \quad \underline{p} \leq p \end{cases},$$

and so firm 1 is willing to randomize on $\left[\underline{p}, \infty\right)$. The verification for firm 2 is identical. ■

A.2 Lemma 3.2 Proof

Proof. Consider an arbitrary equilibrium and let $v \geq 0$ be the infimum of the support of firm 1's distribution over prices. Naturally, then, $v + \lambda$ must be the infimum of the support of firm 2's distribution over prices. Thus, the consumer's net payoff is weakly less than $\max\{y - v - \lambda, x - v\} = x - v \leq x$. ■

A.3 Proposition 3.3 Proof

Proof. By Lemma 3.2, the consumer's payoff at any (x, y) is bounded above by $\min\{x, y\}$, which is weakly concave. For $\kappa > 0$ this function is strictly concave. ■

A.4 Propositions 4.2 and 4.3 Proofs

Proof. We are looking for a symmetric equilibrium in which each firm chooses an atomless distribution over prices $\Gamma(p)$ with support on $\left[\underline{p}, \underline{p}+2\lambda\right]$. We guess further that Γ can be written as $\Gamma(p)=\Gamma_L(p)$ for $p \in \left[\underline{p}, \underline{p}+\lambda\right]$ and $\Gamma(p)=\Gamma_H(p)$ for $p \in \left[\underline{p}+\lambda, \underline{p}+2\lambda\right]$. The profit for a firm is

$$\Pi(p)=\frac{p}{n}\left[(n-1)(1-\Gamma(p))^{n-2}(1-\Gamma(p+\lambda))+\left(1-\Gamma(p-\lambda)^{n-1}\right)\right],$$

or

$$\Pi(p)=\begin{cases}\frac{p}{n}\left[(n-1)(1-\Gamma_L(p))^{n-2}(1-\Gamma_H(p+\lambda))+1\right], & \underline{p} \leq p \leq \underline{p}+\lambda \\ \frac{p}{n}(1-\Gamma_L(p-\lambda))^{n-1}, & \underline{p}+\lambda \leq p \leq \underline{p}+2\lambda\end{cases}.$$

For any on-path p a firm's payoff must equal some constant $\frac{k}{n}$. Thus, for all $p \in [\tilde{p}, \bar{p}]$, we have

$$\frac{p}{n}(1-\Gamma_L(p-\lambda))^{n-1}=\frac{k}{n},$$

which we can rearrange to get

$$\Gamma_L(p)=1-\left(\frac{k}{p+\lambda}\right)^{\left(\frac{1}{n-1}\right)}.$$

Since $\Gamma_L\left(\underline{p}\right)=0$, we must have $k=\underline{p}+\lambda$.

Next, for all $p \in \left[\underline{p}, \tilde{p}\right]$, we must have (substituting in for Γ_L and k)

$$\frac{\underline{p}+\lambda}{n}=\frac{p}{n}\left[(n-1)\left(\frac{\underline{p}+\lambda}{p+\lambda}\right)^{\left(\frac{n-2}{n-1}\right)}(1-\Gamma_H(p+\lambda))+1\right],$$

Some rearranging yields

$$\Gamma_H(p)=1-\frac{\underline{p}+2\lambda-p}{(p-\lambda)(n-1)}\left(\frac{p}{\underline{p}+\lambda}\right)^{\left(\frac{n-2}{n-1}\right)}$$

Furthermore, $\Gamma_L\left(\underline{p}+\lambda\right)=\Gamma_H\left(\underline{p}+\lambda\right)$ or

$$\frac{\lambda}{\left(\underline{p}\right)(n-1)}=\left(\frac{\underline{p}+\lambda}{\underline{p}+2\lambda}\right)^{\left(\frac{1}{n-1}\right)},$$

which pins down $\underline{p}$. Let us guess that $\underline{p} = \beta\lambda$ for some $\beta > 0$. This implies

$$I(\beta) := \left(\frac{1+\beta}{2+\beta}\right)^{\left(\frac{1}{n-1}\right)} - \frac{1}{\beta(n-1)} = 0 \; .$$

I is strictly increasing in β and is strictly negative for all β sufficiently small and strictly positive for all β sufficiently large. Thus, our guess is correct. I has a unique root, which is strictly decreasing in n. Accordingly, $\underline{p}$ is strictly decreasing in n and as $n \uparrow \infty$, $\underline{p} \downarrow 0$. Moreover, as $n \uparrow \infty$, $\Gamma_L\left(\underline{p}+\lambda\right) \downarrow 0$. Moreover, Γ_H becomes steeper and steeper as n increases. Thus, $\Gamma \to \delta_\lambda$.

Finally, we need to verify that firms do not want to choose a price outside of the conjectured region. If a firm chooses a price $p \in \left[\underline{p}-\lambda, \underline{p}\right]$, its payoff is

$$\frac{p}{n}\left[(n-1)(1-\Gamma_L(p+\lambda))+1\right] = \frac{p}{n}\left[(n-1)\left(\frac{\underline{p}+\lambda}{p+2\lambda}\right)^{\left(\frac{1}{n-1}\right)}+1\right] .$$

The derivative of this with respect to p is

$$\frac{((n-2)p+2\lambda(n-1))\left(\frac{\underline{p}+\lambda}{p+2\lambda}\right)^{\left(\frac{1}{n-1}\right)}}{p+2\lambda}+1>0 \; ,$$

whence we conclude a firm does not want to deviate to a price in this region (we have implicitly assumed that $\underline{p} \geq \lambda$, but this is fine since a firm obviously does not want to deviate to a negative price). Evidently, if a firm chooses any price $p \leq \underline{p}-\lambda$ its payoff is just p, which is obviously strictly increasing in p and hence equals $\underline{p}+\lambda$, which we just established is not an improvement for the firm. The last case is that in which a firm chooses a price $p \in \left[\underline{p}+2\lambda, \underline{p}+3\lambda\right]$. In that case, a firm's profit is

$$\frac{p}{n}(1-\Gamma_H(p-\lambda))^{n-1} = \frac{p}{n}\left(\frac{\underline{p}+3\lambda-p}{(p-2\lambda)(n-1)}\right)^{n-1}\left(\frac{p-\lambda}{\underline{p}+\lambda}\right)^{(n-2)} ,$$

which is strictly decreasing in p.

The uniqueness argument is analogous to that argued for the atomless equilibrium of Proposition 3 in Moscarini and Ottaviani (2001). ■

A.5 Proposition 4.5 Proof

Proof. If one firm, firm 2, say, chooses Φ, firm 1's profit as a function of p is $(1-q)\frac{q}{1-2q}\lambda$, a constant, for all $p \in \left[\frac{q}{1-2q}\lambda, \frac{q}{1-2q}\lambda+\lambda\right]$. For all $p \in \left[\frac{q}{1-2q}\lambda+\lambda, \frac{q}{1-2q}\lambda+2\lambda\right]$, firm 1's profit is

$$pq\left(1-\Phi\left(p-\lambda\right)\right),$$

which is strictly decreasing in p. For all $p \geq \frac{q}{1-2q}\lambda+2\lambda$ firm 1's profit is 0. Finally, for all $p \in \left[0, \frac{q}{1-2q}\lambda+\lambda\right]$, firm 1's profit is $p\left(1-q\Phi\left(p+\lambda\right)\right)$. For all

$$q \leq \frac{\frac{\sqrt[3]{9\sqrt{93}-47}}{\sqrt[3]{2}} - \frac{11\sqrt[3]{2}}{\sqrt[3]{9\sqrt{93}-47}} + 5}{9} \approx .406,$$

this function is strictly increasing on this interval. ∎

A.6 Theorem 5.1 Proof

Proof. By symmetry we restrict attention WLOG to the case $y \geq x$. We assume that the firms each choose the distributions over prices F with support on $\left[\underline{p}, \bar{p}\right]$. Defining $\lambda := \bar{p}-\underline{p}$, the consumer's payoff from posterior (x, y), is

$$V\left(x,y\right) = \begin{cases} y-\mathbb{E}\left[p\right]-\kappa c\left(x,y\right), & y \geq x+\lambda \\ y-\mathbb{E}\left[p\right]+U\left(z\right)-\kappa c\left(x,y\right), & x+\lambda \geq y \geq x \end{cases}, \tag{♉}$$

where $z := y-x$ and

$$\mathbb{E}\left[p\right] = \int_{\underline{p}}^{\underline{p}+\lambda} p dF\left(p\right),$$

and

$$U\left(z\right) := \int_{\underline{p}+z}^{\underline{p}+\lambda} \left(p-z\right)F\left(p-z\right)dF\left(p\right) - \int_{\underline{p}}^{\underline{p}+\lambda-z} p\left(1-F\left(p+z\right)\right)dF\left(p\right).$$

Directly,

$$V_{xx}\left(x,y\right) = \begin{cases} -\kappa c_{xx}\left(x,y\right), \\ \int_{\underline{p}+z}^{\underline{p}+\lambda} f\left(p-z\right)dF\left(p\right)-\kappa c_{xx}\left(x,y\right) \end{cases} \qquad V_{yy}\left(x,y\right) = \begin{cases} -\kappa c_{yy}\left(x,y\right), \\ \int_{\underline{p}+z}^{\underline{p}+\lambda} f\left(p-z\right)dF\left(p\right)-\kappa c_{yy}\left(x,y\right) \end{cases},$$

and

$$V_{xy} = -\int_{\underline{p}+z}^{\underline{p}+\lambda} f(p-z)\,dF(p) - \kappa c_{xy}(x,y).$$

The directional second derivative in the direction of $(1,1)$ is $-\kappa c_{xx}(x,y) - \kappa c_{yy}(x,y) - 2\kappa c_{xy}(x,y) < 0$, by the strict convexity of c. As the price function on the triangle Δ^1 (♃) is a plane the support of the learning must be on the line $y = 2\mu - x$ and so we conclude that the consumer only learns along the line $y = 2\mu - x$. ■

A.7 Theorem 5.2 Proof

Proof. For convenience, define $\underline{p} := \sqrt{2}\lambda$, $\tilde{p} := \underline{p} + \lambda$, and $\bar{p} := \underline{p} + 2\lambda$. The consumer's payoff as a function of the realized posterior belief (x,y) is (restricting attention to $y \geq x$ by symmetry)

$$V(x,y) = \begin{cases} y - \mathbb{E}[p] - \kappa c(x,y), & y \geq x + 2\lambda \\ y - \mathbb{E}[p] + T_1(z) - \kappa c(x,y), & x + 2\lambda \geq y \geq x + \lambda \\ y - \mathbb{E}[p] + T_2(z) - \kappa c(x,y), & x + \lambda \geq y \geq x \end{cases}, \qquad (♆)$$

where $z := y - x$ and

$$\mathbb{E}[p] = \int_{\underline{p}}^{\tilde{p}} p\,d\Gamma_L(p) + \int_{\tilde{p}}^{\bar{p}} p\,d\Gamma_L(p) = \left(\left(\sqrt{2}+1\right)\log\left(\sqrt{2}+1\right) + \sqrt{2} - 1\right)\lambda\,,$$

$$T_1(z) := \int_{\underline{p}}^{\bar{p}-z} \left(1 - \Gamma_H(p+z)\right)\Gamma_L(p)\,dp\,,$$

and

$$T_2(z) := \int_{\tilde{p}}^{\bar{p}-z} \left(1 - \Gamma_H(p+z)\right)\Gamma_H(p)\,dp + \int_{\tilde{p}-z}^{\tilde{p}} \left(1 - \Gamma_H(p+z)\right)\Gamma_L(p)\,dp + \int_{\underline{p}}^{\tilde{p}-z} \left(1 - \Gamma_L(p+z)\right)\Gamma_L(p)\,dp\,.$$

From Theorem 5.1, we may restrict attention to learning along the line $y = 2\mu - x$. The directional derivative along vector $(1,-1)$, evaluated at all points of the form $(x, 2\mu - x)$, is

$$D(x) = \begin{cases} -\kappa c_x(x, 2\mu - x) + \kappa c_y(x, 2\mu - x) - 1, & x \leq \mu - \lambda \\ 2P_1(2\mu - 2x) - \kappa c_x(x, 2\mu - x) + \kappa c_y(x, 2\mu - x) - 1, & \mu - \lambda \leq x \leq \mu - \frac{\lambda}{2} \\ 2P_2(2\mu - 2x) - \kappa c_x(x, 2\mu - x) + \kappa c_y(x, 2\mu - x) - 1, & \mu - \frac{\lambda}{2} \leq x \leq \mu \end{cases}, \qquad (♇)$$

where

$$P_1(z) := \int_{\underline{p}}^{\bar{p}-z} \gamma_H(p+z)\Gamma_L(p)\,dp\ ,$$

and

$$P_2(z) := \int_{\tilde{p}}^{\bar{p}-z} \gamma_H(p+z)\Gamma_H(p)\,dp + \int_{\tilde{p}-z}^{\tilde{p}} \gamma_H(p+z)\Gamma_L(p)\,dp + \int_{\underline{p}}^{\tilde{p}-z} \gamma_L(p+z)\Gamma_L(p)\,dp\ .$$

Direct substitution yields $D(\mu) = 0$. Moreover, by the symmetry and convexity of c, and since $c(\mu,\mu) = 0$, for $x \le \mu$, $c_y(x, 2\mu - x) - c_x(x, 2\mu - x) \ge 0$ with equality at $x = \mu$.

When κ is large or when ω is sufficiently large, we need $D\left(\mu - \frac{\lambda}{2}\right) = D\left(\mu + \frac{\lambda}{2}\right) = 0$, i.e.,

$$\tau(\kappa,\lambda) := 2P_1(\lambda) - \kappa c_x\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) + \kappa c_y\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) - 1 = 0. \qquad (\boldsymbol{\Theta})$$

Note that

$$P_1(\lambda) = \int_{\underline{p}}^{\tilde{p}} \gamma_H(p+\lambda)\Gamma_L(p)\,dp = -\frac{\left(2^{\frac{5}{2}}+6\right)\log\left(\sqrt{2}+2\right) - \left(2^{\frac{7}{2}}+12\right)\log\left(\sqrt{2}+1\right) + \left(2^{\frac{3}{2}}+3\right)\log(2) + 2}{2},$$

which is evidently independent of the parameters and is approximately $\frac{1}{10}$. Directly, $\tau'(\kappa) > 0$ and

$$\tau'(\lambda) = \frac{\kappa}{2}\left[c_{xx}\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) - 2c_{xy}\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) + c_{yy}\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right)\right] > 0.$$

By the implicit function theorem $\lambda'(\kappa) < 0$. Moreover, $\lim_{\lambda \to \min\{2\mu, 2(1-\mu)\}} \tau = \infty$ and $\tau(\kappa, 0) < 0$. Accordingly, there is a unique solution $\lambda^* = \lambda(\kappa)$ to this equation, which is strictly decreasing in κ. Moreover, $\lim_{\kappa\uparrow\infty} \lambda^*(\kappa) = 0$ and $\lim_{\kappa\downarrow 0} \lambda^*(\kappa) = 2\min\{\mu, 1-\mu\}$. We also need to check the following:

Claim A.1. *$D(x) \le 0$ for all $x \in [\mu - \lambda^*, \mu]$; and $V\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) \ge V(\mu,\mu)$.*

Proof. Directly, we differentiate the function D (from Expression ♀) with respect to x. This yields

$$D'(x) = \underbrace{-\kappa c_{xx}(x, 2\mu - x) - \kappa c_{yy}(x, 2\mu - x) + 2\kappa c_{xy}(x, 2\mu - x)}_{=:\kappa\rho(x)<0} + \tau(x),$$

where

$$\tau(x) = \begin{cases} 0, & x \le \mu - \lambda \\ 4R(2\mu - x), & \mu - \lambda \le x \le \mu - \frac{\lambda}{2} \\ 4M(2\mu - x), & \mu - \frac{\lambda}{2} \le x \le \mu \end{cases},$$

where

$$R(z) := \int_{\underline{p}}^{\bar{p}-z} \gamma_H(p+z)\gamma_L(p)\,dp\ ,$$

and

$$M(z) := \int_{\tilde{p}}^{\bar{p}-z} \gamma_H(p+z)\gamma_H(p)\,dp + \int_{\tilde{p}-z}^{\tilde{p}} \gamma_H(p+z)\gamma_L(p)\,dp + \int_{\underline{p}}^{\tilde{p}-z} \gamma_L(p+z)\gamma_L(p)\,dp\ .$$

It is straightforward to check that $\tau(x) \ge 0$ (strictly if $x > \mu - \lambda$) and that it is strictly increasing in x (for all $x > \mu - \lambda$). Moreover, $\rho'(x) = c_{yyy} - c_{xxx} + 3c_{xxy} - 3c_{yyx} \le 0$, by assumption. Accordingly, the second derivative has at most one sign change, from negative to positive. Given the zero slope condition at $x = \mu - \frac{\lambda^*}{2}$, this establishes the claim. ∎

For this distribution to be feasible (a fusion of the prior) we need $\frac{\lambda^*}{2} \le \omega$. Define $\bar{\kappa} \ge 0$ to be the value of κ such that $\lambda^* = 2\omega$. Observe that $\bar{\kappa} = 0$ if and only if $\omega = \max\{1-\mu, \mu\}$, which are the special cases in which at most one product is low value or at most one product is high value, respectively. Directly, $D(\mu - \omega)$ is strictly increasing in κ.

Claim A.2. *There exists an interval of* κ*s,* $[\underline{\kappa}, \bar{\kappa}]$*, where* $\underline{\kappa} \in [0, \bar{\kappa}]$ *for which the equilibrium* $\lambda = 2\omega$.

Proof. Directly, $\frac{\partial}{\partial \kappa} D(x) > 0$. By construction, when $\kappa = \bar{\kappa}$, the equilibrium $\lambda^* = 2\omega$. Moreover, as we noted in Claim A.1, $V\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) \ge V(\mu, \mu)$. If this is an equality then $\underline{\kappa} = \bar{\kappa}$. If this inequality is strict then, by the intermediate value theorem, there is an interval of κs ($[\underline{\kappa}, \bar{\kappa}]$) for which line tangent to $V(x - \omega, x + \omega)$ lies above $V(x, 2\mu - x)$ at μ. ∎

∎

A.8 Proposition 5.4

Proof. Thanks to the discussion in the text, we need only consider the case $\kappa \geq \bar{\kappa}$. In this case, the consumer's payoff at equilibrium is

$$\underbrace{\mu + \frac{\lambda}{2} - \mathbb{E}[p] + \int_{\underline{p}}^{\tilde{p}} (1 - \Gamma_H(p+\lambda))\Gamma_L(p)\,dp}_{S(\lambda)} - \kappa c\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right).$$

The derivative of this with respect to λ is $S'(\lambda) - \frac{1}{2}\kappa\left(c_y\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) - c_x\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right)\right)$. However, by the Optimality Equation ʘ, $c_y\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) - c_x\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) = 2P_1(\lambda) - 1$. Summing everything up and simplifying, we obtain $-1 - \sqrt{2} < 0$. ∎

A.9 Theorem 5.5 and Corollary 5.6 Proofs

Proof. For convenience, define $\underline{p} := \frac{q}{1-2q}\lambda$. The consumer's payoff as a function of the realized posterior belief x is (restricting attention to $y \geq x$ by symmetry)

$$V(x,y) = \begin{cases} y - \mathbb{E}[p] - \kappa c(x,y), & y \geq x + \lambda \\ y - \mathbb{E}[p] + U(z) - \kappa c(x,y), & x + \lambda \geq y \geq x \end{cases},$$

where $z := y - x$ and

$$\mathbb{E}[p] = \int_{\underline{p}}^{\underline{p}+\lambda} p\,d\Phi(p)\ ,$$

and

$$U(z) := \int_{\underline{p}+z}^{\underline{p}+\lambda} (p-z)\Phi(p-z)\,d\Phi(p) - \int_{\underline{p}}^{\underline{p}+\lambda-z} p(1-\Phi(p+z))\,d\Phi(p)\ .$$

For this to be an equilibrium, we need for there to be a line $\alpha x + \beta$ lying everywhere above $V(x, 2\mu - x)$ on $0 \leq x \leq \mu$, and intersecting $V(x, 2\mu - x)$ at $\mu - \frac{\lambda}{2}$ and μ. Removing $\mathbb{E}[p]$ since it is a constant, we compute

$$V(\mu,\mu) = \mu - \frac{\lambda(1-q)q\left(\log\left(\frac{q}{1-q}\right) - 4q + 2\right)}{(1-2q)^3}.$$

Moreover, $\alpha = \kappa c_y\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) - \kappa c_x\left(\mu - \frac{\lambda}{2}, \mu + \frac{\lambda}{2}\right) - 1$.

We need

$$\left(\kappa c_y\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right)-\kappa c_x\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right)-1\right)\left(\mu-\frac{\lambda}{2}\right)+\beta=\mu+\frac{\lambda}{2}-\kappa c\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right),$$

or

$$\beta=2\mu-\kappa c\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right)-\left(\kappa c_y\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right)-\kappa c_x\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right)\right)\left(\mu-\frac{\lambda}{2}\right).$$

We also need

$$\left(\kappa c_y\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right)-\kappa c_x\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right)-1\right)\mu+\beta=\mu-\frac{\lambda(1-q)q\left(\log\left(\frac{q}{1-q}\right)-4q+2\right)}{(1-2q)^3},$$

or

$$\beta=2\mu-\frac{\lambda(1-q)q\left(\log\left(\frac{q}{1-q}\right)-4q+2\right)}{(1-2q)^3}-\left(\kappa c_y\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right)-\kappa c_x\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right)\right)\mu.$$

Equating the βs, we get

$$\underbrace{\omega\frac{(1-q)\left(\log\left(\frac{q}{1-q}\right)-4q+2\right)}{(1-2q)^3}}_{=:t(\lambda)}+\kappa\underbrace{\left[\lambda d'(\lambda)-d(\lambda)\right]}_{v(\lambda)}=0, \tag{$\mathfrak{D}$}$$

where

$$d(\lambda):=c\left(\mu-\frac{\lambda}{2},\mu+\frac{\lambda}{2}\right),$$

and where we used the fact that $t\equiv t(\lambda)$ (as $q=\frac{\omega}{\lambda}$).

Furthermore, note that we must have $2\omega\le\lambda\le 2\max\{\mu,1-\mu\}$. Directly, $t'(\lambda)<0$ and $t(\lambda)<0$ for all $\lambda\in[2\omega,2\max\{\mu,1-\mu\}]$, and $\lim_{\lambda\downarrow 2\omega}t(\lambda)=0$. Moreover, by the strict convexity of c, $v(\lambda)>0$. Likewise, $v'(\lambda)=\lambda d''(\lambda)>0$, and $\lim_{\lambda\uparrow 2\max\{\mu,1-\mu\}}v'(\lambda)=\infty$. Continuing along these lines, it is easy to compute that $t'(\lambda)$ is bounded for all $\lambda\in(2\omega,2\max\{\mu,1-\mu\}]$. Finally, it is straightforward to check that $\lim_{\lambda\uparrow 2\max\{\mu,1-\mu\}}v(\lambda)=\infty$.

From the observations in the previous paragraph, we conclude the following:

(i) If κ is sufficiently small, then a unique solution $\lambda^*(\kappa)$ to Equation $\mathfrak{D}$ exists.

(ii) In this unique solution, λ^* is strictly decreasing in κ.

(iii) As $\kappa\downarrow 0$, $\lambda^*\uparrow 2\max\{\mu,1-\mu\}$.

This last item is the stated corollary (5.6). ■