# When Does the Waterbed Effect Harm Consumers?

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Buyer power can produce a waterbed, where a powerful buyer pays less and its rival more. Does that harm consumers? When the discounted buyer is larger, its price reduction reaches more consumers. If retail prices respond more to own costs than rival costs, a one-for-one waterbed benefits consumers. Raising the rival's cost shifts sales from the supplier's higher-margin customer, so Hotelling, Cournot, and Bertrand do not even generate one-for-one. Buyer power therefore benefits consumers. The harm condition in Inderst and Valletti (2011) is never met. Under asymmetric Bertrand, harm requires the locally *smaller* buyer to receive the discount.

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## 1 Introduction

When a supermarket chain grows large enough to negotiate deep discounts from its suppliers, the worry is that suppliers recoup the lost margin by charging smaller rivals more. The wholesale price rises on one end because it fell on the other—the "waterbed effect." Smaller rivals then face higher input costs than the chain does, may pass some of that through to consumers, and lose share. The chain's bargaining gains may spill over into higher prices at competing checkouts. But

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does the waterbed harm consumers?

Following Inderst and Valletti (2011), an incumbent supplier chooses per-unit wholesale offers to a multi-market chain and its independent rivals; each buyer can switch to another supplier by paying a fixed cost. The chain and its rivals accept the supplier's offers and continue to sell, with the offers leaving each just indifferent between accepting and switching. We leave the downstream game unspecified until the applications. The waterbed comes from changing the rival's outside option. Because the chain spreads the fixed cost of switching across more outlets, the supplier must offer it a discount to keep it accepting the offer. At the same time, a rival that switches still competes against the now-cheaper chain, so switching becomes less attractive for the rival. This allows the supplier to charge the rival more, since the rival's outside option worsened.

The wholesale waterbed is not consumer harm. Whether consumers lose depends on how much the rival's cost rises and how retail prices respond. Under local symmetry, the discounted chain becomes the larger seller. If an equal fall in the chain's cost and rise in the rival's cost lowers the chain's retail price at least as much as it raises the rival's, the price reduction reaches more sales than the price increase. A one-for-one waterbed therefore cannot reduce consumer surplus.

Can the supplier choose a more than one-for-one waterbed that could then generate consumer harm? That question depends on the downstream game. A larger surcharge moves sales away from the rival, where the incumbent supplier earns the higher margin, toward the chain, where it earns the lower one. The supplier must also keep both retailers willing to accept its offers. I prove that in the Hotelling, Cournot, and Bertrand applications, the supplier never chooses a wholesale response as large as one-for-one. Consumer surplus therefore rises on the margin even if the rival's retail price rises. Inderst and Valletti (2011)'s Proposition 6 gives a sufficient condition for the waterbed to reduce consumer surplus. The one-for-one bound rules out that condition.

Existing waterbed models usually assume that the two retailers are otherwise symmetric within each local market; the chain is large because it owns many outlets (Inderst and Valletti 2011; Bedre-Defolie and Caprice 2011; Prasad 2012). Once local demand differs, however, a chain can own many outlets and still sell less than its rival in a particular market. If the rival remains the larger seller, the supplier has more units over which to collect the higher wholesale price. The resulting retail-price increase also reaches more consumers, while the chain's discount reaches fewer. In asymmetric Bertrand, marginal harm requires the discounted firm to sell less than its rival in that market (Theorem 2). A chain can therefore be large overall and locally smaller, but that is not the Walmart example. The FTC's since-dismissed 2025 complaint against PepsiCo alleged that Walmart received promotional pricing and marketing support unavailable to smaller retailers (Fruits 2026, p. 8). In the usual Walmart example, the chain is both the powerful buyer and the larger seller in the local market. The theorem rules out marginal consumer harm in that configuration.

The waterbed concern has circulated in competition policy for more than two decades. Dobson and Inderst (2008) organized the informal arguments for how the waterbed could arise. The UK Competition Commission examined a waterbed theory in its 2008 groceries market investigation and declined to rely on it, finding the evidence insufficient (Competition Commission 2008, §§5.19–5.43). The OECD treated the effect as a standing concern in its 2008 roundtable on monopsony and buyer power (OECD 2009, p. 11) and returned to it in its 2022 roundtable on purchasing power (OECD 2022, p. 14). The CMA revisited waterbed-type arguments in Sainsbury's/Asda (Competition and Markets Authority 2019, §§15.26–15.46).

Related work also shows why it is difficult to move from a wholesale waterbed to consumer harm. The wholesale effect itself depends on contracting. With two-part tariffs, Chen (2003) finds that greater buyer power lowers the fringe's wholesale price, and Bedre-Defolie and Caprice (2011) show that the response depends on contract form and the supplier's cost function. Downstream competition can also work against the waterbed (King 2013). In homogeneous Cournot, greater retailer size raises customer welfare despite the waterbed (Prasad 2012).

Section 2 sets up the model. Section 3 establishes the wholesale waterbed and derives the consumer-harm threshold. Section 4 proves the main impossibility theorem. Section 5 verifies its conditions in three benchmark applications. Section 6 applies the results to Inderst and Valletti's Proposition 6. Section 7 derives the local-size condition under asymmetric demand.

# 2 Model

A single upstream supplier with constant marginal cost $k \geq 0$ sells to downstream firms through per-unit wholesale contracts. There are $N$ identical local markets. In each market, two downstream firms compete. Each firm $i$ has processing cost $c \geq 0$. If it accepts wholesale offer $w_i$, its gross marginal cost is $m_i := c + w_i$.

Let $j$ denote firm $i$'s rival. Until Section 7, the two firms have the same downstream profit, price, and quantity functions under relabeling. For any pair of marginal costs, write these functions as $\pi(m_i, m_j)$, $p(m_i, m_j)$, and $q(m_i, m_j)$, with own cost as the first argument. When firm labels matter, $p_i = p(m_i, m_j)$ and $q_i = q(m_i, m_j)$. Write $\partial_r \pi$ and $\partial_{rs} \pi$ for derivatives with respect to the indicated cost arguments.

**Assumption 1.** The downstream profit function $\pi$ is twice continuously differentiable and satisfies

$$\partial_1 \pi < 0, \qquad \partial_2 \pi > 0, \qquad \partial_{12} \pi < 0.$$

The cross-partial means that the profit gain from a rival's higher cost is smaller when the firm's own cost is higher. It captures the business-stealing force behind the waterbed: when one firm becomes cheaper, its rival's outside option deteriorates relative to accepting the incumbent supplier's offer.

A large buyer $L$ controls $n_L$ outlets, with $2 \leq n_L \leq N$, each facing an independent small rival $S$. Write $w^0$ for the common wholesale price at the symmetric benchmark and $m^0 := c + w^0$. In the other $N - n_L$ markets, both firms remain independent, so their wholesale price remains $w^0$. A buyer that rejects the incumbent supplier's offer accesses an alternative source at gross marginal cost $m^A := c + k$. Each independent firm pays a fixed switching cost $F > 0$ to do so, while $L$ pays that cost once across its outlets. Let $\lambda \in (0, 1]$ denote the share of $F$ borne by $L$ in each local market, so its effective switching cost per market is $\lambda F$. At the symmetric benchmark, $\lambda = 1$. The post-merger value is $\lambda = 1/n_L$, because $L$ pays the switching cost once across its $n_L$ outlets. For the merger comparison, $\lambda$ varies over $[1/n_L, 1]$. A lower value of $\lambda$ represents stronger buyer power.

The supplier moves first and makes simultaneous, publicly observable take-it-or-leave-it offers to all downstream firms. The ex ante identical small rivals receive a common offer $w_S \geq k$, and $L$ receives $w_L \geq k$ for its outlets. After observing both offers, buyers simultaneously accept or reject, and downstream competition then takes place. Indifference at the acceptance stage is resolved in favor of acceptance. We write $S$ for a representative small rival and restrict identical small rivals to use the same pure acceptance strategy.

Working backward from downstream competition, consider the buyers' acceptance decisions. Conditional on rival cost $x_j$, define each buyer's gain from acceptance by

$$G_S(w_S; x_L) := \pi(m_S, x_L) - \pi(m^A, x_L) + F,$$
$$G_L(w_L; x_S; \lambda) := \pi(m_L, x_S) - \pi(m^A, x_S) + \lambda F.$$

Thus buyer $i$ accepts when $G_i \geq 0$ and rejects when $G_i < 0$. Under the symmetric-strategy restriction, the acceptance stage has four possible outcomes:

| | | |
|---|---|---|
| $L$ and the small rivals accept | $G_L(w_L; m_S; \lambda) \geq 0$ | $G_S(w_S; m_L) \geq 0,$ |
| only $L$ accepts | $G_L(w_L; m^A; \lambda) \geq 0$ | $G_S(w_S; m_L) < 0,$ |
| only the small rivals accept | $G_L(w_L; m_S; \lambda) < 0$ | $G_S(w_S; m^A) \geq 0,$ |
| neither accepts | $G_L(w_L; m^A; \lambda) < 0$ | $G_S(w_S; m^A) < 0.$ |

If an offer pair admits more than one equilibrium of the acceptance stage, the buyers' acceptance strategies after that offer pair specify which one is played. Beyond the symmetry restriction and the tie-breaking convention, we impose no further equilibrium-selection rule. Anticipating these acceptance decisions and downstream competition, the supplier chooses its offers to maximize total profit. Given the buyers' acceptance strategies, a *supplier optimum* is an offer pair that globally maximizes supplier profit over all possible offers. A pure-strategy subgame-perfect equilibrium specifies a supplier optimum, the buyers' acceptance decisions after every offer pair, and downstream equilibrium play. In the $n_L$ markets served by $L$, if $L$ alone accepts, the incumbent earns $n_L(w_L - k)q_L(m_L, m^A)$. If only the small rivals accept, it earns $n_L(w_S - k)q_S(m_S, m^A)$. If neither $L$ nor the small rivals accept, it earns zero from these markets.

We study equilibria in which $L$ and all of its small rivals accept, sell positive quantities, and receive offers that leave their participation constraints binding. If $G_L(w_L; m_S; \lambda) \geq 0$ and $G_S(w_S; m_L) \geq 0$, $L$ and the small rivals accept in the unique equilibrium of the acceptance stage. Because $w_i \geq k$ and $\partial_{12}\pi < 0$, accepting the incumbent's offer rather than switching is weakly more attractive when the rival rejects than when the rival accepts. The two inequalities therefore make acceptance weakly dominant, and the tie-breaking rule selects it. Thus equilibrium selection does not affect acceptance at the offers studied below. It can still affect the supplier's profit from a deviation and hence whether an offer pair is a supplier optimum.

Because markets are independent and the supplier has constant marginal cost, changing $\lambda$ does not affect its offers in the other $N - n_L$ markets. Their contribution to supplier profit is therefore constant. Conditional on $L$ and the small rivals accepting, the supplier chooses $w_L$ and $w_S$ to maximize profit in the $n_L$ markets served by $L$:

$$\begin{aligned} \max_{w_L, w_S \geq k} \quad & \Pi(w_L, w_S) := n_L \Big[ (w_L - k) q_L(m_L, m_S) + (w_S - k) q_S(m_S, m_L) \Big] \\ \text{subject to} \quad & G_L(w_L; m_S; \lambda) \geq 0, \qquad G_S(w_S; m_L) \geq 0. \end{aligned} \tag{1}$$

A solution to (1) is an equilibrium offer pair only if the supplier cannot earn more by deviating to offers followed by a different acceptance outcome.

Both participation constraints bind.

$$G_S(w_S; m_L) = \pi(m_S, m_L) - \pi(m^A, m_L) + F = 0, \tag{2}$$

$$G_L(w_L; m_S; \lambda) = \pi(m_L, m_S) - \pi(m^A, m_S) + \lambda F = 0. \tag{3}$$

We next ask how the offers that leave both buyers indifferent change as $\lambda$ falls.

Let $J$ denote the Jacobian of $(G_S, G_L)$ with respect to $(m_S, m_L)$ at such an offer pair:

$$J := \begin{pmatrix} \partial_1 \pi(m_S, m_L) & \partial_2 \pi(m_S, m_L) - \partial_2 \pi(m^A, m_L) \\ \partial_2 \pi(m_L, m_S) - \partial_2 \pi(m^A, m_S) & \partial_1 \pi(m_L, m_S) \end{pmatrix}.$$

**Assumption 2.** There are continuously differentiable functions $(m_S(\lambda), m_L(\lambda))$ on $\lambda \in [1/n_L, 1]$ that satisfy (2)–(3) and $m_S(1) = m_L(1) = m^0$. At every $\lambda$ in this interval, both firms sell positive quantities and $\det J > 0$.

We use these functions for the comparative statics below and write $w_i(\lambda) = m_i(\lambda) - c$ for the associated offers. At every $\lambda$, the offers leave both buyers indifferent between accepting and switching. They need not maximize supplier profit at every value of $\lambda$. At an equilibrium, the offers must also maximize supplier profit given the buyers' acceptance strategies. Because switching is costly and profit falls with own cost, binding participation requires $m_L, m_S > m^A$, so $w_L, w_S > k$ and the lower bounds in (1) are slack.

# 3 From the Wholesale Waterbed to Consumer Harm

## 3.1 The wholesale-level waterbed

**Proposition 1.** *Under Assumptions 1 and 2, the offers $w_L(\lambda)$ and $w_S(\lambda)$ defined above satisfy $w_L < w^0 < w_S$ for every $\lambda \in [1/n_L, 1)$.*

*Proof.* At the symmetric endpoint, both buyers receive the same offer. It therefore suffices to determine how the two offers that leave them indifferent change with $\lambda$. Differentiating the participation constraints (2)–(3) gives

$$\frac{dm_L}{d\lambda} = \frac{-F\partial_1 \pi(m_S, m_L)}{\det J} > 0, \qquad \frac{dm_S}{d\lambda} = -\frac{F[\partial_2 \pi(m^A, m_L) - \partial_2 \pi(m_S, m_L)]}{\det J} < 0.$$

The first derivative is positive because $\partial_1 \pi < 0$ and $\det J > 0$. For the second, binding participation and $F > 0$ imply $m_S > m^A$, while $\partial_{12} \pi < 0$ implies $\partial_2 \pi(m^A, m_L) > \partial_2 \pi(m_S, m_L)$. Thus an increase in $\lambda$ raises $w_L$ and lowers $w_S$. Moving from the symmetric endpoint $\lambda = 1$ to any $\lambda < 1$ gives $w_L < w^0 < w_S$. ■

Intuitively, lowering $\lambda$ requires a lower $w_L$ to keep $L$ indifferent between accepting and switching. This worsens $S$'s outside option relative to accepting, so $w_S$ must rise to keep $S$ indifferent as well. Both offers move in these directions throughout the interval, although the size of the adjustment may vary. The more sharply $L$'s discount reduces $S$'s profit from switching relative to accepting, the more the supplier must raise $S$'s offer. The wholesale waterbed hurts the rival buyer. Whether it hurts consumers depends on how both firms' prices respond and how many consumers each price change reaches.

## 3.2 The consumer-level waterbed condition

For the remainder of this section, assume $p_S(m_S, m_L)$ is continuously differentiable with $\partial p_S/\partial m_S > 0$ and $\partial p_S/\partial m_L > 0$. The first says that $S$'s price rises with its own cost; the second says that it also rises with $L$'s cost. These price responses are separate restrictions on downstream competition and do not follow from the profit restrictions in Assumption 1. They hold in all three downstream models of Section 5.

Among nearby offer pairs that leave both buyers indifferent, a lower $w_L$ requires a higher $w_S$.[1] Implicit differentiation of $S$'s participation constraint (2) gives

$$\frac{dw_S}{dw_L} = -\frac{\partial_2 \pi(m_S, m_L) - \partial_2 \pi(m^A, m_L)}{\partial_1 \pi(m_S, m_L)} < 0. \tag{4}$$

When both buyers are indifferent, $S$'s participation constraint determines $w_S$ as a function of $w_L$, while $L$'s constraint determines the corresponding $\lambda$. The numerator in (4) is negative because $m_S > m^A$ and $\partial_{12}\pi < 0$; the denominator is negative because $\partial_1 \pi < 0$. For these nearby offers, define the waterbed response

$$\rho := -\frac{dm_S}{dm_L} = -\frac{dw_S}{dw_L} > 0.$$

Whether $p_S$ also rises is a separate question because it responds to both input

1. The neighboring offers need not maximize supplier profit.

costs. As $w_L$ falls,

$$\frac{dp_S}{dw_L} = \frac{\partial p_S}{\partial m_L} + \frac{\partial p_S}{\partial m_S} \cdot \frac{dw_S}{dw_L}. \tag{5}$$

The first term is positive because $S$'s price rises with $L$'s cost. A reduction in $L$'s cost therefore pulls $S$'s retail price down. The second term is negative. The wholesale-level waterbed raises $w_S$, pushing $S$'s retail price up. The consumer-level waterbed bites if and only if the second force dominates the first.

**Lemma 1.** *Maintain Assumptions 1 and 2. Suppose $p_S$ is continuously differentiable and*

$$\frac{\partial p_S}{\partial m_S} > 0, \qquad \frac{\partial p_S}{\partial m_L} > 0.$$

*As $\lambda$ varies while both participation constraints remain binding, $S$'s retail price rises when $w_L$ falls ($dp_S/dw_L < 0$) if and only if*

$$\left|\frac{dw_S}{dw_L}\right| > \frac{\partial p_S/\partial m_L}{\partial p_S/\partial m_S}. \tag{6}$$

*Proof.* Equation (5) gives $dp_S/dw_L < 0$ if and only if

$$\frac{\partial p_S}{\partial m_L} < -\frac{\partial p_S}{\partial m_S}\frac{dw_S}{dw_L}.$$

Dividing by the positive own-cost response $\partial p_S/\partial m_S$ gives (6). ∎

The threshold $(\partial p_S/\partial m_L)/(\partial p_S/\partial m_S)$ is a property of downstream competition alone. It measures the rival-cost response in $S$'s retail price as a fraction of the own-cost response. In Hotelling, this ratio is $1/2$: $\partial p_S/\partial m_L = 1/3$ and $\partial p_S/\partial m_S = 2/3$.

### 3.3 The consumer-harm threshold

Whether $S$'s retail price rises is not the welfare question. Consumer surplus depends on both firms' prices and sales, so the condition for consumer harm differs from the condition for $S$'s price to rise. For $i, j \in \{L, S\}$, let $\tau_{ij} := \partial p_i/\partial m_j$ denote the response of firm $i$'s retail price to firm $j$'s cost, and let $s_i := q_i/(q_L + q_S)$ denote firm $i$'s local sales share. Thus

$$\tau_{SL} = \frac{\partial p_S}{\partial m_L}, \quad \tau_{SS} = \frac{\partial p_S}{\partial m_S}, \quad \tau_{LL} = \frac{\partial p_L}{\partial m_L}, \quad \tau_{LS} = \frac{\partial p_L}{\partial m_S}.$$

**Proposition 2.** *Consider nearby offer pairs that leave both buyers indifferent. Suppose both firms sell positive quantities, $p_L$ and $p_S$ are continuously differentiable, consumers have quasilinear preferences, and all four price responses $\tau_{ij}$ are positive. Define*

$$\rho^* := \frac{q_L \tau_{LL} + q_S \tau_{SL}}{q_L \tau_{LS} + q_S \tau_{SS}}. \tag{7}$$

*Then*

$$\frac{d\mathrm{CS}}{dm_L} = (q_L \tau_{LS} + q_S \tau_{SS})(\rho - \rho^*). \tag{8}$$

*Consumer surplus falls when $m_L$ falls (i.e. $d\mathrm{CS}/dm_L > 0$) if and only if $\rho > \rho^*$.*

*Proof.* With quasilinear preferences, a marginal increase in a firm's price reduces consumer surplus by that firm's sales. Thus $\partial\mathrm{CS}/\partial p_i = -q_i$ and

$$d\mathrm{CS} = -q_L\, dp_L - q_S\, dp_S.$$

For nearby offers that leave both buyers indifferent, $dm_S/dm_L = -\rho$, so

$$\frac{dp_L}{dm_L} = \tau_{LL} - \rho\, \tau_{LS}, \qquad \frac{dp_S}{dm_L} = \tau_{SL} - \rho\, \tau_{SS}.$$

Therefore,

$$\begin{aligned}\frac{d\mathrm{CS}}{dm_L} &= -q_L(\tau_{LL} - \rho\, \tau_{LS}) - q_S(\tau_{SL} - \rho\, \tau_{SS}) \\ &= -(q_L \tau_{LL} + q_S \tau_{SL}) + \rho(q_L \tau_{LS} + q_S \tau_{SS}) \\ &= (q_L \tau_{LS} + q_S \tau_{SS})(\rho - \rho^*).\end{aligned}$$

Because $q_L \tau_{LS} + q_S \tau_{SS} > 0$, consumer surplus falls when $m_L$ falls if and only if $\rho > \rho^*$. ■

A fall in $m_L$ benefits consumers when the price effects of $L$'s discount outweigh those of the rival-cost increase. Consumer surplus falls only when the wholesale response exceeds the sales-weighted harm threshold. In the symmetric benchmarks below, that threshold equals one at symmetry and is at least one once $L$ becomes the larger seller; the inequality is strict except under homogeneous Cournot. A one-for-one rival-cost response is therefore harmless.

# 4 When Consumer Harm Is Impossible

Theorem 1 requires two restrictions on downstream competition. First, each firm's own cost has at least as strong an effect on its retail price as its rival's cost does, and this difference is no smaller for $L$. Second, the firms face the same local demand and respond to costs in the same way.

**Assumption 3.** For the marginal costs $(m_S(\lambda), m_L(\lambda))$ in Assumption 2, $p_L$ and $p_S$ are continuously differentiable, all four price responses are positive, and

$$\tau_{LL} - \tau_{LS} \;\geq\; \tau_{SS} - \tau_{SL} \;\geq\; 0.$$

In the symmetric Hotelling, linear Cournot, and linear Bertrand applications below, pass-through is constant and symmetry gives $\tau_{LL} = \tau_{SS}$ and $\tau_{LS} = \tau_{SL}$. Assumption 3 therefore reduces to each firm's retail price responding at least as much to its own cost as to its rival's cost. The inequality is strict in Hotelling and differentiated Cournot and Bertrand, and holds with equality in homogeneous Cournot.

**Assumption 4** (Symmetric downstream competition)**.** The two firms have the same reduced-form profit, price, and quantity functions under relabeling. Each firm's equilibrium quantity is strictly decreasing in its own marginal cost and strictly increasing in its rival's marginal cost.

The discount makes $L$ cheaper and, under symmetry, the larger seller. Assumption 3 then prevents an equal fall in $L$'s cost and rise in $S$'s cost from harming consumers. Consumer harm therefore requires the rival's cost to rise by more than $L$'s cost falls.

**Theorem 1.** *Maintain Assumptions 1–4 and suppose consumers have quasilinear preferences. If $n_L \geq 2$, then at every $\lambda \in [1/n_L, 1)$,*

$$-\frac{dm_S}{dm_L} \leq 1 \quad \Longrightarrow \quad \frac{d\text{CS}}{d\lambda} \leq 0.$$

*If the rival's cost rises by no more than L's cost falls, then a marginal increase in buyer power does not reduce consumer surplus.*

*Proof.* We show that the consumer-harm threshold is at least one. At any such point with $n_L \geq 2$, Proposition 1 gives $m_L < m_S$—the waterbed regime—so the large buyer has the cost advantage. Quantity monotonicity gives

$$q_L = q(m_L, m_S) > q(m_S, m_S) > q(m_S, m_L) = q_S.$$

Relative to $S$, $L$ has both a lower own cost and a higher-cost rival. Both differences increase $L$'s sales.

Since $q_L > q_S$ and $\tau_{LL} - \tau_{LS} \geq \tau_{SS} - \tau_{SL} \geq 0$ by Assumption 3,

$$\begin{aligned}
&(q_L\tau_{LL} + q_S\tau_{SL}) - (q_L\tau_{LS} + q_S\tau_{SS}) \\
&\qquad = q_L(\tau_{LL} - \tau_{LS}) - q_S(\tau_{SS} - \tau_{SL}) \\
&\qquad \geq (q_L - q_S)(\tau_{SS} - \tau_{SL}) \geq 0,
\end{aligned}$$

so $\rho^* \geq 1$.

If $-dm_S/dm_L = \rho \leq 1$, then $\rho \leq \rho^*$, so Proposition 2 gives $d\text{CS}/dm_L \leq 0$. Proposition 1 gives $dm_L/d\lambda > 0$. Therefore $d\text{CS}/d\lambda \leq 0$: a marginal reduction in $\lambda$ weakly raises consumer surplus. ∎

Theorem 1 separates downstream competition from the supplier's choice of offers. Under symmetry, the discount makes $L$ the larger local seller. Under Assumption 3, consumers cannot lose on the margin when $S$'s cost rises by no more than $L$'s cost falls. Equivalently, marginal harm requires a more-than-one-for-one increase in $S$'s cost. The applications show that supplier profit maximization prevents that response in the three benchmark models. A different upstream setting can produce the larger response. In Majumdar (2005), a merger reduces the demand available to an upstream entrant, which must recover a fixed entry cost from fewer sales. The wholesale response can then exceed one-for-one and harm consumers.

Theorem 1 applies once buyer power has created a cost difference. At the symmetric benchmark, the firms have equal costs and sales, and the consumer-harm threshold equals one. The next result shows that the wholesale response is strictly below one there. Thus the first marginal increase in buyer power raises consumer surplus without imposing Assumption 3.

**Corollary 1.** *Maintain Assumptions 1, 2, and 4. Suppose $p_L$ and $p_S$ are continuously differentiable, consumers have quasilinear preferences, and all four price responses are positive. As $\lambda$ falls marginally from the symmetric endpoint $\lambda = 1$ and the offers adjust to keep both buyers indifferent, consumer surplus rises.*

*Proof.* At the symmetric endpoint, the consumer-harm threshold equals one. We show that the wholesale response is strictly smaller. Symmetry gives $q_L = q_S$, $\tau_{LL} = \tau_{SS}$, and $\tau_{LS} = \tau_{SL}$, so the numerator and denominator of $\rho^*$ coincide and are positive.

At the symmetric endpoint,

$$\det J = [-\partial_1 \pi(m^0, m^0)]^2 - [\partial_2 \pi(m^A, m^0) - \partial_2 \pi(m^0, m^0)]^2 > 0.$$

Both bracketed terms are positive, so the first exceeds the second. Equation (4) therefore gives

$$\rho = \frac{\partial_2 \pi(m^A, m^0) - \partial_2 \pi(m^0, m^0)}{-\partial_1 \pi(m^0, m^0)} < 1.$$

Proposition 2 implies $d\mathrm{CS}/dm_L < 0$. Finally, $dm_L/d\lambda = -F\partial_1 \pi(m^0, m^0)/\det J > 0$, so $d\mathrm{CS}/d\lambda < 0$ at $\lambda = 1$. A marginal reduction in $\lambda$ therefore raises consumer surplus. ∎

The corollary starts from the symmetric benchmark $\lambda = 1$, where combining the outlets provides no switching-cost advantage. A small fall in $\lambda$ introduces that advantage. The result does not compare consumer surplus before and after a discrete merger.

# 5 Standard Downstream Models

Hotelling, differentiated Cournot, and differentiated Bertrand satisfy the downstream assumptions. We verify the conditions model by model and then impose supplier profit maximization. The appendix shows that when both buyers sell and are indifferent between accepting and switching, optimal offers keep the rival-cost response below one.

## 5.1 Hotelling

Under Hotelling competition with transport cost $t$:

$$\pi(m_i, m_j) = \frac{1}{2t}\left[t + \frac{m_j - m_i}{3}\right]^2, \qquad p_i(m_i, m_j) = t + \frac{2m_i + m_j}{3}.$$

The price derivatives are constant: $\tau_{SL} = \tau_{LS} = 1/3$ and $\tau_{SS} = \tau_{LL} = 2/3$. The two own–cross gaps are equal, $\tau_{LL} - \tau_{LS} = \tau_{SS} - \tau_{SL} = 1/3 > 0$. Assumption 3 holds.

The harm threshold is

$$\rho^* = \frac{q_L \cdot 2/3 + q_S \cdot 1/3}{q_L \cdot 1/3 + q_S \cdot 2/3} = \frac{2 - s_S}{1 + s_S},$$

where full coverage gives $s_S = q_S$. At the symmetric benchmark, $s_S = 1/2$ and $\rho^* = 1$; throughout the waterbed regime $s_S < 1/2$, so $\rho^* > 1$.

The wholesale pass-through follows from (4) with $\partial_1 \pi = -(3t + m_j - m_i)/(9t)$, $\partial_2 \pi = (3t + m_j - m_i)/(9t)$, and $\partial_{12}\pi = -1/(9t)$:

$$\frac{dw_S}{dw_L} = -\frac{w_S - k}{6t \cdot s_S}, \tag{9}$$

where $s_S = 1/2 + (m_L - m_S)/(6t)$. For any offer pair with $s_S > 0$, define $\widehat{\rho}_H := (w_S - k)/(6t\, s_S)$. When both buyers are indifferent, differentiating $S$'s participation constraint gives equation (9) and identifies $\widehat{\rho}_H$ with the wholesale pass-through $\rho$. Equation (9) is equation (9) in Inderst and Valletti (2011). Lemma 3 (Appendix A.1) shows that $\widehat{\rho}_H = \rho < 1$ at every equilibrium in the strict waterbed regime where both firms have positive market shares and both buyers are indifferent between accepting and switching. This includes $\lambda = 1/n_L$ for every finite $n_L \geq 2$.

With full coverage, total demand is inelastic, so consumer surplus depends on how the price burden is distributed across firms and market shares. At every equilibrium in the strict waterbed regime where both firms have positive market shares and both participation constraints bind, Lemma 3 gives $\rho < 1 < \rho^*$. Marginal consumer harm is therefore impossible.

## 5.2 Cournot

In each local market, the two downstream firms compete in quantities. Each firm $i$ faces inverse demand $p_i = a - q_i - \gamma q_j$, where $\gamma \in (0, 1]$ indexes substitutability. Goods are homogeneous when $\gamma = 1$ and differentiated when $\gamma < 1$. Set $c = k = 0$ for simplicity. The Cournot equilibrium gives:

$$q_i = \frac{2(a - m_i) - \gamma(a - m_j)}{4 - \gamma^2}, \qquad p_i(m_i, m_j) = \frac{a(2 - \gamma) + (2 - \gamma^2)m_i + \gamma m_j}{4 - \gamma^2}.$$

Since $p_i - m_i = q_i$ from the first-order condition, profit is $\pi(m_i, m_j) = q_i^2$. Assumption 1 is satisfied: $\partial_{12}\pi = -4\gamma/(4 - \gamma^2)^2 < 0$, and the remaining conditions hold whenever quantities are positive.

The price pass-throughs are constant, with $\tau_{LL} = \tau_{SS} = (2 - \gamma^2)/(4 - \gamma^2)$ and $\tau_{SL} = \tau_{LS} = \gamma/(4 - \gamma^2)$. Both own–cross gaps equal

$$\tau_{LL} - \tau_{LS} = \frac{(2 + \gamma)(1 - \gamma)}{4 - \gamma^2} = \frac{1 - \gamma}{2 - \gamma} > 0$$

for $\gamma \in (0, 1)$. Assumption 3 holds strictly for imperfect substitutes; the gap closes to zero at $\gamma = 1$ (homogeneous goods). At the homogeneous endpoint $\gamma = 1$, all four pass-throughs equal $1/3$, so Assumption 3 holds with equality and $\rho^* = 1$. Lemma 4 gives $\rho < 1$, so Proposition 2 gives $d\text{CS}/dm_L < 0$. Prasad (2012, Proposition 4.3) proves the corresponding no-harm result under homogeneous Cournot. Our result extends the marginal conclusion to differentiated Cournot and isolates the supplier's less-than-one-for-one response.

The pass-through from (4) is:

$$\frac{dw_S}{dw_L} = -\frac{\gamma\, w_S}{(4-\gamma^2)\, q_S}. \tag{10}$$

For any offer pair with $q_S > 0$, define $\widehat{\rho}_C := \gamma w_S / [(4-\gamma^2) q_S]$. When both buyers are indifferent, differentiating $S$'s participation constraint gives equation (10) and identifies $\widehat{\rho}_C$ with $\rho$.

At the symmetric benchmark, $|dw_S/dw_L| = \gamma w^0 / [(2-\gamma)(a-w^0)] < 1$ by Corollary 1, so the waterbed is less than one-for-one and consumer surplus is marginally increasing in buyer power. Away from the symmetric benchmark, Lemma 4 (Appendix A.2) shows that $\rho < 1$ at every equilibrium in the waterbed regime where both firms sell positive quantities and both participation constraints bind. Hence $\rho < \rho^*$ and marginal harm is impossible at every such equilibrium.

## 5.3 Bertrand

In each local market, the two downstream firms compete in prices. Each firm $i$ faces direct demand $q_i = a - p_i + \gamma p_j$, where $\gamma \in (0,1)$ indexes substitutability. Set $c = k = 0$. The Bertrand equilibrium gives:

$$p_i = \frac{a(2+\gamma) + 2m_i + \gamma m_j}{4-\gamma^2}, \qquad q_i = \frac{a(2+\gamma) - (2-\gamma^2) m_i + \gamma m_j}{4-\gamma^2}.$$

The first-order condition $q_i = p_i - m_i$ gives profit $\pi(m_i, m_j) = q_i^2$, as in Cournot. Assumption 1 is satisfied: $\partial_{12}\pi = -2\gamma(2-\gamma^2)/(4-\gamma^2)^2 < 0$.

The equilibrium pass-throughs are constant, with $\tau_{LL} = \tau_{SS} = 2/(4-\gamma^2)$ and $\tau_{SL} = \tau_{LS} = \gamma/(4-\gamma^2)$. Both own–cross gaps equal

$$\tau_{LL} - \tau_{LS} = \frac{2-\gamma}{4-\gamma^2} = \frac{1}{2+\gamma} > 0.$$

Assumption 3 holds.

The pass-through from (4) is:

$$\frac{dw_S}{dw_L} = -\frac{\gamma\, w_S}{(4-\gamma^2)\, q_S}, \tag{11}$$

the same functional form as the Cournot pass-through (10). As in Cournot, the marginal effect favors consumers at the symmetric benchmark. At the post-merger value $\lambda = 1/n_L$, Lemma 5 (Appendix A.3) shows $\rho < 1$ at every equilibrium in the waterbed regime at which both buyers sell positive quantities and are indifferent between accepting and switching. Thus $\rho < 1 < \rho^*$ at every such equilibrium, and marginal harm is impossible.

The three applications give the paper's main no-harm result.

**Proposition 3.** *Maintain Assumptions 1 and 2, and suppose consumers have quasilinear preferences. Consider full-coverage Hotelling, linear Cournot with $\gamma \in (0,1]$, or differentiated linear Bertrand at the post-merger value $\lambda = 1/n_L$. If $w_L(1/n_L)$ and $w_S(1/n_L)$ are the equilibrium offers, then $d\mathrm{CS}/dm_L < 0$.*

The proof is in Appendix A. In each model, the discounted buyer sells more, and retail prices respond at least as much to own costs as to rival costs. Supplier profit maximization keeps the rival's cost increase below the level needed to offset the consumer gain. Table 1 compares the wholesale response required for consumer harm with the response allowed by supplier profit maximization.

Table 1: Marginal harm at post-merger equilibria when both buyers sell positive quantities and have binding participation constraints

| Model | Consumer-harm threshold | Wholesale response in equilibrium | Marginal harm? |
|---|---|---|---|
| Hotelling | $\rho^* > 1$ | $\rho < 1$ | No |
| Cournot ($\gamma \in (0,1]$) | $\rho^* \geq 1$ | $\rho < 1$ | No |
| Bertrand | $\rho^* > 1$ | $\rho < 1$ | No |

Notes: $\rho^*$ is the consumer-harm threshold from Proposition 2; marginal harm requires $\rho > \rho^*$. The wholesale response $\rho$ is the increase in $S$'s cost per unit decrease in $L$'s cost. The supplier bounds follow from Lemmas 3–5. For Cournot, the harm threshold equals one only when $\gamma = 1$.

Hotelling fixes total demand, whereas Cournot and Bertrand allow it to respond, so the result does not depend on inelastic demand. Retail prices respond at least as much to own costs as to rival costs, and supplier profit maximization keeps the wholesale response below one-for-one.

# 6 Application to Inderst–Valletti's Proposition 6

**Proposition 4.** *Fix $F > 0$. At every full-coverage Hotelling equilibrium with finite $n_L \geq 2$ in which both firms have positive market shares, both buyers are indifferent between accepting and switching, and $w_S > w_L$,*

$$\rho < 1 < \rho^*, \qquad \frac{d\mathrm{CS}}{dm_L} < 0.$$

*Proof.* Lemma 3 gives $\rho < 1$ at every finite-$n_L$ equilibrium in the proposition. Because $w_S > w_L$, Hotelling demand gives $s_S < 1/2$. Section 5.1 gives $\rho^* = (2 - s_S)/(1 + s_S) > 1$. Proposition 2 therefore implies $d\mathrm{CS}/dm_L < 0$. ■

This result applies directly to Inderst and Valletti's Proposition 6, which gives a sufficient condition for a marginal increase in buyer power to harm consumers. They write the small firm's market share as $y_S$. Substituting $y_S = s_S$, their condition (12) is

$$\frac{w_S - k}{3t} > \frac{2s_S(2 - s_S)}{1 + s_S}.$$

Dividing by $2s_S > 0$ gives

$$\frac{w_S - k}{6t\, s_S} > \frac{2 - s_S}{1 + s_S}.$$

When both buyers are indifferent between accepting and switching, equation (9) identifies the left side with $\rho$, while Section 5.1 identifies the right side with $\rho^*$. Their condition is therefore $\rho > \rho^*$. Proposition 4 establishes the opposite ordering at every equilibrium in its domain, so the condition is never satisfied there. Inderst and Valletti's Proposition 6 remains a valid implication, but it yields no consumer-harm conclusion at these equilibria. Consumer surplus rises on the margin even when the rival's retail price rises. In a short companion note, I prove the same vacuity result directly in Inderst and Valletti's notation (Albrecht 2026).

# 7 What Marginal Consumer Harm Requires

Symmetry links cost and local size. Because $L$ pays less, it sells more. With asymmetric demand, $L$ can pay less and still be the smaller local seller. The result below asks whether this reversal is necessary for marginal harm at a post-merger equilibrium where both firms sell and both participation constraints bind.

## 7.1 Asymmetric Bertrand

Keep differentiated Bertrand, but let the two firms in a market face different demand intercepts. We maintain quasilinear preferences; the demands below are generated by a quadratic utility:

$$q_L = a_L - p_L + \gamma p_S, \qquad q_S = 1 - p_S + \gamma p_L, \qquad \gamma \in (0,1),$$

with the rival's intercept normalized to one and $a_L > 0$ denoting $L$'s demand intercept. Set $c = k = 0$ and write $\delta := 4 - \gamma^2$. The Bertrand equilibrium gives $\pi_i = q_i^2$ and

$$\delta\, q_L = (2a_L + \gamma) - (2 - \gamma^2)\, m_L + \gamma\, m_S, \qquad \delta\, q_S = (2 + \gamma a_L) + \gamma\, m_L - (2 - \gamma^2)\, m_S.$$

The symmetric model of Section 5.3 is the case $a_L = 1$. The own-cost-dominance inequality holds for every $a_L$: the pass-throughs are the same constants as in the symmetric case, $\tau_{LL} = \tau_{SS} = 2/\delta$ and $\tau_{LS} = \tau_{SL} = \gamma/\delta$.

The supplier still chooses both offers to maximize profit subject to each buyer's participation constraint, but quantities and profits are now firm-specific. The acceptance gains $G_i$ are defined as before using $\pi_i$. Because retail-price responses remain symmetric, the harm threshold exceeds one exactly when $L$ sells more than $S$. The theorem shows that a marginal increase in buyer power cannot harm consumers in equilibrium when $q_L \geq q_S$.

**Theorem 2.** *Consider asymmetric Bertrand with $\gamma \in (0,1)$, $a_L > 0$, and $n_L \geq 2$. Fix any equilibrium at $\lambda = 1/n_L$ where both firms sell positive quantities and both participation constraints bind. If $q_L \geq q_S$, a marginal fall in $\lambda$ across nearby offer pairs that leave both buyers indifferent raises consumer surplus. Hence marginal consumer harm requires $q_L < q_S$.*

The proof is in Appendix B. The restriction concerns local quantity share, not the number of outlets. A multi-market buyer can still be smaller in a given market.

# 8 Conclusion

The waterbed effect is a claim about rivals, not consumers. A supplier may respond to a powerful buyer's discount by charging the buyer's rivals more, and those rivals may raise their retail prices. But the same discount lowers the powerful buyer's cost and shifts sales toward it. Raising the rival's wholesale price also pushes business away from the supplier's higher-margin customer. In Hotelling, Cournot, and differentiated Bertrand, that tradeoff keeps the rival's cost increase smaller than the buyer's cost reduction. Consumers gain even while the rival pays and charges more.

The usual policy narrative points in the wrong direction. It begins with a large powerful buyer winning the discount. Yet the asymmetric Bertrand theorem requires the discounted buyer to be locally smaller before the waterbed can harm consumers. The theorem therefore rules out harm in the familiar case of a powerful buyer that is also the larger local seller.

Dobson and Inderst raised the prospect of consumer harm nearly two decades ago (Dobson and Inderst 2008, p. 333). Inderst and Valletti then made it central to their waterbed model (Inderst and Valletti 2011, pp. 1–2). Later work repeated that interpretation (Escrihuela-Villar and Ferrarese 2021, p. 87), and the OECD continued to treat consumer harm as a policy concern (OECD 2022, p. 14). Yet the equilibrium error went uncorrected. Inderst and Valletti's consumer-harm condition cannot hold in equilibrium. Waterbed claims therefore deserve skepticism, not a presumption of consumer harm.

# A Proof of no harm in canonical downstream models

The model-specific proofs below use the supplier's constrained pricing conditions. The following lemma records those conditions. Fix Hotelling, linear Cournot, or symmetric or asymmetric Bertrand. Suppose $F > 0$ and $\lambda > 0$, and consider an equilibrium where both firms sell positive quantities and both participation constraints bind. Here $\Pi$ denotes the supplier's profit from the affected markets, and $G_i$ denotes buyer $i$'s participation gain when both buyers accept.

**Lemma 2.** *At the equilibrium offers, $w_L > k$ and $w_S > k$. There are nonnegative participation multipliers $\mu_L$ and $\mu_S$ such that*

$$\frac{\partial \Pi}{\partial w_i} + \mu_L \frac{\partial G_L}{\partial w_i} + \mu_S \frac{\partial G_S}{\partial w_i} = 0, \qquad i \in \{L, S\}.$$

*Proof.* Write $\pi_i$ for firm $i$'s reduced-form profit (so $\pi_i = \pi$ in the symmetric models). Binding participation gives

$$\pi_S(m_S, m_L) - \pi_S(m^A, m_L) = -F < 0, \qquad \pi_L(m_L, m_S) - \pi_L(m^A, m_S) = -\lambda F < 0.$$

Since profit falls with own cost, $m_i > m^A$ and hence $w_i > k$. For the symmetric models,

$$\frac{\partial G_i}{\partial w_i} = \partial_1 \pi(m_i, m_j) < 0, \qquad \frac{\partial G_i}{\partial w_j} = \partial_2 \pi(m_i, m_j) - \partial_2 \pi(m^A, m_j) < 0,$$

by $\partial_1 \pi < 0$, $m_i > m^A$, and $\partial_{12} \pi < 0$. In asymmetric Bertrand, $\pi_i = q_i^2$. When both firms sell positive quantities,

$$\partial_1 \pi_i = -\frac{2\nu q_i}{\delta} < 0, \qquad \partial_{12} \pi_i = -\frac{2\gamma\nu}{\delta^2} < 0,$$

where $\nu = 2 - \gamma^2$ and $\delta = 4 - \gamma^2$, so the same inequalities hold. A small decrease in both wholesale prices strictly relaxes the two participation constraints and leaves the lower bounds slack. The usual constrained first-order conditions therefore hold at the supplier's equilibrium offers, with the nonnegative participation multipliers stated in the lemma. ■

## A.1 Hotelling one-for-one bound

Under Hotelling with full coverage ($v$ large), all consumers buy. Consumer surplus is

$$\mathrm{CS} = v - p_L\, s_L - p_S\, s_S - \frac{t}{2}(s_L^2 + s_S^2),$$

where $s_L + s_S = 1$. Because the consumer at the market boundary is indifferent between the two firms, a marginal shift in that boundary has no first-order effect on consumer surplus. As $m_L$ changes and $m_S$ adjusts to keep $S$ indifferent between accepting and switching,

$$\frac{d\mathrm{CS}}{dm_L} = -\frac{dp_L}{dm_L}\, s_L - \frac{dp_S}{dm_L}\, s_S = -\frac{1}{3}\big[(2-\rho)\, s_L + (1-2\rho)\, s_S\big],$$

where $\rho = |dw_S/dw_L| = (w_S - k)/(6t\, s_S)$ is the waterbed pass-through magnitude. For an arbitrary offer pair with $s_S > 0$, write $\widehat{\rho}_H := (w_S - k)/(6t\, s_S)$. When both buyers are indifferent, differentiating $S$'s participation constraint identifies $\widehat{\rho}_H$ with $\rho$. Consequently, a reduction in $m_L$ raises consumer surplus if and only if $\rho < (1 + s_L)/(2 - s_L)$. At the symmetric benchmark the threshold is 1, and $\rho < 1$ there by Corollary 1.

Away from the symmetric benchmark, supplier optimization imposes a one-for-one bound on the waterbed response.

**Lemma 3.** *In full-coverage Hotelling, fix $F > 0$ and $\lambda \in (0, 1)$. Every equilibrium with positive market shares, $w_S > w_L$, and both participation constraints binding satisfies $\widehat{\rho}_H < 1$. In particular, the bound holds at $\lambda = 1/n_L$ for every finite $n_L \geq 2$.*

*Proof.* Normalize the offers by $3t$: let $\widetilde{w}_S := (w_S - k)/(3t)$ and $\widetilde{w}_L := (w_L - k)/(3t)$, so the shares are $s_S = (1 - \widetilde{w}_S + \widetilde{w}_L)/2$ and $s_L = (1 + \widetilde{w}_S - \widetilde{w}_L)/2$, and $\widehat{\rho}_H = \widetilde{w}_S/(2s_S)$. In these variables, $\widehat{\rho}_H \leq 1$ is equivalent to $\widetilde{w}_S \leq 2s_S$.

The strict waterbed regime has $\widetilde{w}_S > \widetilde{w}_L \geq 0$. The strict inequality is equivalent to $s_S < 1/2$. At the equilibrium in the lemma, both participation constraints bind. Lemma 2 gives $\widetilde{w}_L, \widetilde{w}_S > 0$ and nonnegative participation multipliers. After rescaling these multipliers, the supplier's pricing conditions are

$$2s_S\,\mu_S + \widetilde{w}_L\,\mu_L = 2s_S - \tfrac{1}{2}, \qquad \widetilde{w}_S\,\mu_S + 2s_L\,\mu_L = 2s_L - \tfrac{1}{2}, \tag{12}$$

with $\mu_S, \mu_L \geq 0$. The first condition forces $s_S \geq 1/4$. The two binding participation constraints also give

$$(\widetilde{w}_S - \widetilde{w}_L)(2 - \widetilde{w}_S - \widetilde{w}_L) = \frac{2F}{t}(1-\lambda) > 0,$$

so $\widetilde{w}_S + \widetilde{w}_L < 2$.

Eliminating the multipliers in (12) gives

$$\begin{aligned}\mu_S\,\mathcal{D}_H &= \tfrac{1}{2}\big[1 - \widetilde{w}_S(3-4s_S)\big],\\ \mu_L\,\mathcal{D}_H &= \tfrac{1}{2}\big[1 - \widetilde{w}_L(4s_S-1)\big],\\ \mathcal{D}_H &:= 4s_S s_L - \widetilde{w}_S\widetilde{w}_L.\end{aligned}$$

Suppose $\widetilde{w}_S \geq 2s_S$. If $s_S = 1/4$, the first condition in (12) forces both multipliers to be zero, contradicting the second condition. Thus $s_S \in (1/4, 1/2)$. On this interval, $2s_S(3-4s_S) > 1$, so $\widetilde{w}_S(3-4s_S) > 1$ and $\mu_S\mathcal{D}_H < 0$, which forces $\mathcal{D}_H < 0$. Then $\mu_L\mathcal{D}_H \leq 0$ requires $\widetilde{w}_L(4s_S-1) \geq 1$. Because $4s_S - 1 < 1$, this gives $\widetilde{w}_L > 1$. Hence $\widetilde{w}_S > \widetilde{w}_L > 1$ and $\widetilde{w}_S + \widetilde{w}_L > 2$, contradicting $\widetilde{w}_S + \widetilde{w}_L < 2$. Therefore $\widetilde{w}_S < 2s_S$.

■

At every equilibrium in the strict waterbed regime where both firms have positive market shares and both participation constraints bind, $\rho < 1 < (2-s_S)/(1+s_S) = \rho^*$, so consumer surplus is marginally increasing in buyer power.

## A.2 Cournot one-for-one bound

In Cournot, the binding participation constraints alone do not limit the wholesale response enough to rule out consumer harm. The needed restriction comes from the supplier's choice of $w_S$.

Inverse demand is $p_i = a - q_i - \gamma q_j$ with $\gamma \in (0, 1]$; set $a = 1$ and $c = k = 0$. Equilibrium quantities and profits are as in Section 5.2.

At the symmetric benchmark, $\rho < 1$ by Corollary 1. For an arbitrary offer pair with $q_S > 0$, write $\widehat{\rho}_C := \gamma w_S/[(4-\gamma^2)q_S]$. When both buyers are indifferent, differentiating $S$'s participation constraint identifies $\widehat{\rho}_C$ with $\rho$. Away from the symmetric benchmark, supplier optimization gives the needed bound.

**Lemma 4.** *In the linear Cournot model with $\gamma \in (0,1]$, every equilibrium in the waterbed regime $0 \leq w_L < w_S$ where both firms sell positive quantities and both participation constraints bind satisfies $\widehat{\rho}_C = \rho < 1$.*

*Proof.* Consider the supplier's marginal incentive to raise $S$'s offer. The participation constraints matter because a higher $w_S$ makes each of them harder to satisfy.

The supplier's per-affected-market profit is $\Pi = w_L\, q(w_L, w_S) + w_S\, q(w_S, w_L)$, so

$$\frac{\partial \Pi}{\partial w_S} = q_S - \frac{2w_S - \gamma w_L}{4-\gamma^2}.$$

Lemma 2 gives nonnegative participation multipliers $\mu_S$ and $\mu_L$. Raising $w_S$ tightens both participation constraints: $\partial G_S/\partial w_S = -4q_S/(4-\gamma^2) < 0$ and $\partial G_L/\partial w_S = -4\gamma w_L/(4-\gamma^2)^2 \leq 0$. The pricing condition for $w_S$ therefore requires $\partial \Pi/\partial w_S \geq 0$, or

$$(4-\gamma^2)\, q_S \;\geq\; 2w_S - \gamma w_L \;>\; \gamma w_S,$$

where the second inequality holds because $\gamma(w_S + w_L) \leq w_S + w_L < 2w_S$. Hence $\widehat{\rho}_C = \gamma w_S/[(4-\gamma^2)\, q_S] < 1$. ■

When both participation constraints bind, $S$'s constraint identifies $\widehat{\rho}_C$ with $\rho$. Because the consumer-harm threshold is at least one, marginal harm is impossible.

## A.3 Bertrand one-for-one bound

Set $c = k = 0$. The differentiated Bertrand model has the linear reduced form

$$q(m_i, m_j) = \frac{\alpha - \beta m_i + \chi m_j}{D}, \qquad \pi = \vartheta q^2,$$

where $(\alpha, \beta, \chi, D, \vartheta) = (a(2+\gamma), 2-\gamma^2, \gamma, 4-\gamma^2, 1)$. Write the cross-to-own cost sensitivity of quantity as

$$\sigma := \frac{\chi}{\beta} = \frac{\gamma}{2-\gamma^2} \in (0,1).$$

**Lemma 5.** *In the differentiated Bertrand model with $\gamma \in (0,1)$, fix $F > 0$ and $n_L \geq 2$, and set $\lambda = 1/n_L$. Every equilibrium in the waterbed regime $0 \leq w_L < w_S$ where both firms sell positive quantities and both participation constraints bind satisfies $\rho < 1$.*

*Proof.* The waterbed ordering gives $q_L > q_S > 0$, so both firms sell positive quantities. Suppose, toward a contradiction, that $\rho \geq 1$. After normalizing, binding participation bounds the two offers, while the supplier's first-order conditions determine the shadow value of $S$'s participation constraint. We show that a one-for-one response would make this shadow value negative.

Normalize $\widetilde{w}_L := \beta w_L/\alpha$, $\widetilde{w}_S := \beta w_S/\alpha$, $\widetilde{q}_S := Dq_S/\alpha = 1 - \widetilde{w}_S + \sigma\widetilde{w}_L$, and $\widetilde{q}_L := Dq_L/\alpha = 1 - \widetilde{w}_L + \sigma\widetilde{w}_S$. Positive sales imply $\widetilde{q}_S, \widetilde{q}_L > 0$, and the pass-through magnitude is $\rho = \chi w_S/(Dq_S) = \sigma\widetilde{w}_S/\widetilde{q}_S$. Thus $\rho < 1$ is equivalent to $\widetilde{q}_S - \sigma\widetilde{w}_S > 0$. Both constraints binding forces $w_L > 0$ (at $w_L = 0$ the large buyer's constraint has slack $\lambda F$), so $0 < \widetilde{w}_L < \widetilde{w}_S$.

Binding participation first restricts the two offers. At the post-merger value, the normalized participation losses (dropping the common positive factor $\vartheta\alpha^2/D^2$) are $\widetilde{\ell}_S = \widetilde{w}_S(2 + 2\sigma\widetilde{w}_L - \widetilde{w}_S)$ and $\widetilde{\ell}_L = \widetilde{w}_L(2 + 2\sigma\widetilde{w}_S - \widetilde{w}_L)$, with $\widetilde{\ell}_L = \lambda\widetilde{\ell}_S$. Expanding, $\widetilde{\ell}_S - \widetilde{\ell}_L = (\widetilde{w}_S - \widetilde{w}_L)(2 - \widetilde{w}_L - \widetilde{w}_S) > 0$, so $\widetilde{w}_L + \widetilde{w}_S < 2$; and $n_L \geq 2$ gives $\widetilde{\ell}_S \geq 2\widetilde{\ell}_L$.

Lemma 2 gives nonnegative participation multipliers. Rescale them as $\widetilde{\mu}_i := (2\beta\vartheta/(n_L D))\mu_i$. The two pricing conditions are

$$\widetilde{q}_S\widetilde{\mu}_S + \sigma\widetilde{w}_L\,\widetilde{\mu}_L = 2\widetilde{q}_S - 1, \qquad \sigma\widetilde{w}_S\,\widetilde{\mu}_S + \widetilde{q}_L\,\widetilde{\mu}_L = 2\widetilde{q}_L - 1, \qquad \widetilde{\mu}_S, \widetilde{\mu}_L \geq 0.$$

The first left side is nonnegative, so $\widetilde{q}_S \geq 1/2$. In particular, $q_S \geq \alpha/(2D)$, so the small buyer remains bounded away from zero sales at such an optimum.

The rival's normalized offer must be below one. Suppose instead that $\widetilde{w}_S \geq 1$. Then $\widetilde{q}_S \geq 1/2$ gives $\sigma\widetilde{w}_L \geq \widetilde{w}_S - 1/2 \geq 1/2$, so $\widetilde{w}_L \geq 1/(2\sigma)$, and $\widetilde{w}_L < \widetilde{w}_S$ with $\widetilde{w}_L + \widetilde{w}_S < 2$ gives $\widetilde{w}_L < 1$, forcing $\sigma > 1/2$. Let $\Delta_\ell(\widetilde{w}_S; \widetilde{w}_L) := \widetilde{\ell}_S - 2\widetilde{\ell}_L$.

Then $\partial\Delta_\ell/\partial\widetilde{w}_S = 2 - 2\widetilde{w}_S - 2\sigma\widetilde{w}_L < 0$ for $\widetilde{w}_S \geq 1$, so $\Delta_\ell(\widetilde{w}_S;\widetilde{w}_L) \leq \Delta_\ell(1;\widetilde{w}_L) = 1-(4+2\sigma)\widetilde{w}_L + 2\widetilde{w}_L^2$; and $\partial\Delta_\ell(1;\widetilde{w}_L)/\partial\widetilde{w}_L = -(4+2\sigma)+4\widetilde{w}_L < 0$ for $\widetilde{w}_L < 1$, so

$$\Delta_\ell(\widetilde{w}_S;\widetilde{w}_L) \;\leq\; \Delta_\ell\Big(1;\frac{1}{2\sigma}\Big) = \frac{1-4\sigma}{2\sigma^2} \;<\; 0 \qquad \text{for } \sigma > \tfrac{1}{2},$$

contradicting $\widetilde{\ell}_S \geq 2\widetilde{\ell}_L$. Hence $0 < \widetilde{w}_L < \widetilde{w}_S < 1$.

The first-order conditions now restrict the multiplier on $S$'s participation constraint. Their determinant is

$$\mathcal{D}_\mu := \widetilde{q}_S\widetilde{q}_L - \sigma^2\widetilde{w}_L\widetilde{w}_S = (1-\widetilde{w}_L)(1-\widetilde{w}_S) + \sigma\widetilde{w}_L(1-\widetilde{w}_L) + \sigma\widetilde{w}_S(1-\widetilde{w}_S) > 0$$

on $0 < \widetilde{w}_L < \widetilde{w}_S < 1$. Eliminating $\widetilde{\mu}_L$ from the two pricing conditions gives

$$\mathcal{S} := \widetilde{q}_L(2\widetilde{q}_S - 1) - \sigma\widetilde{w}_L\,(2\widetilde{q}_L - 1) = \widetilde{\mu}_S\,\mathcal{D}_\mu \;\geq\; 0.$$

It remains to rule out a one-for-one response. The supposition $\rho \geq 1$ is $\widetilde{q}_S \leq \sigma\widetilde{w}_S$. Let $x := (1-\sigma)(\widetilde{w}_L + \widetilde{w}_S)$ and $z := 2\widetilde{q}_S - 1 \geq 0$, so $(1+\sigma)(\widetilde{w}_S - \widetilde{w}_L) = 1-x-z$ and $0 < x < 1-z$. Direct substitution gives $2(1-\sigma^2)(\widetilde{q}_S - \sigma\widetilde{w}_S) = (1-\sigma)[1+(1+2\sigma)z] - 2\sigma^2 x$. Hence $\widetilde{q}_S \leq \sigma\widetilde{w}_S$ requires $x \geq x_c(z) := (1-\sigma)[1+(1+2\sigma)z]/(2\sigma^2)$, and $x_c(z) < 1-z$ requires $z < 2\sigma - 1$ (so again $\sigma > 1/2$). Substituting into $\mathcal{S}$ gives

$$2(1-\sigma^2)\,\mathcal{S} = 4\sigma x^2 + (2\sigma^2 + 4\sigma z - 6\sigma - 2z)\,x + (1-\sigma)(2\sigma + 3z - z^2).$$

The right side is a convex quadratic in $x$. At the lower bound imposed by $\rho \geq 1$ and the limiting upper bound, it becomes

$$\begin{aligned} x = x_c(z):&\quad -\frac{(1-\sigma^2)(2\sigma-1-z)\big[(1-\sigma^2) + z(1+\sigma-\sigma^2)\big]}{\sigma^3} < 0,\\ x = 1-z:&\quad z(1+\sigma)(1+z-2\sigma) \leq 0. \end{aligned}$$

Both signs follow from $z < 2\sigma - 1$. Convexity therefore makes the right side negative for every admissible $x$, so $\mathcal{S} < 0$. Since $\mathcal{D}_\mu > 0$, this would require $\widetilde{\mu}_S < 0$, contrary to the nonnegativity of the multiplier on $S$'s participation constraint. This contradiction proves $\rho < 1$ at every equilibrium in the lemma. ■

The difference between Cournot and Bertrand is how strongly a firm's sales respond to its rival's cost relative to its own cost. In Cournot, the cross-to-own response is $\gamma/2 \leq 1/2$, and the supplier's pricing condition for $w_S$ gives $\rho < 1$ directly.

In Bertrand, the corresponding ratio $\sigma = \gamma/(2-\gamma^2)$ can exceed one-half, so both buyers' indifference between accepting and switching provides the additional restriction. In Hotelling, the cross-to-own response equals one, and Lemma 3 gives the one-for-one bound directly.

*Proof of Proposition 3.* Lemma 2 gives the constrained pricing conditions and non-negative participation multipliers used in the three bounds above. Proposition 1 also gives $w_L < w_S$.

The calculations in Section 5 verify Assumptions 3 and 4 for Hotelling, differentiated Cournot, and Bertrand. When both participation constraints bind, $S$'s constraint identifies $\widehat{\rho}_H = \rho$ in Hotelling and $\widehat{\rho}_C = \rho$ in Cournot. Lemma 3 gives $\rho < 1$ in Hotelling, Lemma 4 gives $\rho < 1$ in differentiated Cournot, and Lemma 5 gives $\rho < 1$ in Bertrand. In each case, Section 5 also gives $\rho^* > 1$. Proposition 2 therefore gives $d\mathrm{CS}/dm_L < 0$.

At the homogeneous Cournot endpoint $\gamma = 1$, all four retail pass-throughs equal $1/3$, so $\rho^* = 1$. Lemma 4 gives $\rho < 1$, and Proposition 2 therefore gives $d\mathrm{CS}/dm_L < 0$. ■

# B Proof of the small-buyer theorem

*Proof of Theorem 2.* Fix an equilibrium described in the theorem and suppose $q_L \geq q_S$. We first rule out a one-for-one rival-cost response. Under that supposition, binding participation requires a sufficiently large gap between the firms' costs. For such a cost gap, the supplier's two pricing conditions require incompatible signs for the participation multipliers.

Let $\nu := 2 - \gamma^2$ and $\delta := 4 - \gamma^2$, and scale quantities by their common Bertrand denominator:

$$Q_L := \delta q_L = (2a_L + \gamma) - \nu m_L + \gamma m_S, \qquad Q_S := \delta q_S = (2 + \gamma a_L) + \gamma m_L - \nu m_S.$$

Thus $Q_L \geq Q_S > 0$. Define

$$b_L := 2Q_L - (2a_L + \gamma), \qquad b_S := 2Q_S - (2 + \gamma a_L).$$

Binding participation and $F > 0$ imply $m_L, m_S > m^A = 0$. Let $\ell_i$ denote buyer $i$'s loss from accepting rather than switching. The quadratic-profit formula gives

$$\delta^2 \ell_i = \nu m_i (2Q_i + \nu m_i).$$

At the post-merger value $\lambda = 1/n_L$, binding participation implies

$$\ell_S = n_L \ell_L \geq 2\ell_L > \ell_L.$$

If $m_S \leq m_L$, then $Q_S \leq Q_L$ would instead imply

$$m_S(2Q_S + \nu m_S) \leq m_L(2Q_L + \nu m_L),$$

or $\ell_S \leq \ell_L$. Hence $m_S > m_L$.

Lemma 2 shows that the supplier's equilibrium offers satisfy the pricing first-order conditions with nonnegative participation multipliers. After rescaling them as $\widetilde{\mu}_i := 2\nu\mu_i / n_L$, those conditions are

$$\gamma m_S \widetilde{\mu}_S + Q_L \widetilde{\mu}_L = \delta b_L, \qquad Q_S \widetilde{\mu}_S + \gamma m_L \widetilde{\mu}_L = \delta b_S. \tag{13}$$

Every term on the left is nonnegative, so $b_L, b_S \geq 0$.

Differentiating $S$'s binding participation constraint gives

$$\rho = -\frac{dm_S}{dm_L} = \frac{\gamma m_S}{Q_S}.$$

Suppose, toward a contradiction, that $\rho \geq 1$, or equivalently $Q_S \leq \gamma m_S$. Measure the cost and quantity gaps in units of $m_L$:

$$r := \frac{m_S - m_L}{m_L} > 0, \qquad h := \frac{Q_L - Q_S}{m_L} \geq 0, \qquad b := \frac{b_S}{m_L} \geq 0, \qquad z := \frac{Q_S}{m_L} > 0.$$

Also let

$$u := \frac{\gamma m_S - Q_S}{m_L} \geq 0.$$

The demand equations and the definitions of $b_L$ and $b_S$ give

$$z = b + (\nu - \gamma) + \nu r, \tag{14}$$

$$\frac{b_L}{m_L} = b + h + (\nu + \gamma) r, \tag{15}$$

$$u = (2\gamma - \nu) - b - (\nu - \gamma) r. \tag{16}$$

Writing

$$\bar{u} := (2\gamma - \nu) - (\nu - \gamma) r = b + u,$$

we have $0 \leq u \leq \bar{u}$.

The participation-loss comparison $\ell_S \geq 2\ell_L$, divided by $m_L^2$, becomes

$$0 \leq 2z(r-1) + \nu(r^2 + 2r - 1) - 4h. \tag{17}$$

We claim that $r > 1/2$. If $r \leq 1/2$, substitute $z = \gamma(1 + r) - u$ into the right side of (17). The expression rises with $u$ and falls with $h$, so $u \leq \bar{u}$ and $h \geq 0$ give the upper bound

$$3\nu r^2 + 2(\nu - \gamma) r + 2\gamma - 3\nu.$$

This quadratic is strictly increasing for $r \geq 0$. At $r = 1/2$, it equals

$$\gamma - \frac{5}{4}\nu < 0,$$

because $\nu > \gamma$ for every $\gamma \in (0, 1)$. This contradicts (17). Hence $r > 1/2$.

We now return to the supplier's pricing conditions. Let

$$\mathcal{D} := \gamma^2 m_L m_S - Q_L Q_S,$$

and define the two expressions that determine the signs of the participation multipliers by

$$N_S := \gamma m_L b_L - Q_L b_S, \qquad N_L := \gamma m_S b_S - Q_S b_L.$$

Because $b, u \geq 0$, $r > 0$, and $\nu - \gamma > 0$, equation (16) implies $2\gamma - \nu > 0$ and

$$b < 2\gamma - \nu < \gamma < \nu + \gamma.$$

Using equations (14)–(16), the first expression can be written as

$$\frac{N_S}{m_L^2} = h(\gamma - b) + \gamma r(\nu + \gamma - b) + bu > 0.$$

The second expression has the opposite sign. In particular,

$$\frac{N_L}{m_L^2} = bu - z\big[h + (\nu + \gamma)r\big] = -E(u) - zh,$$

where

$$E(u) := [\gamma(1 + r) - u](\nu + \gamma)r - (\bar{u} - u)u.$$

For $u \in [0, \bar{u}]$,

$$E'(u) = 2u - \big[(2\gamma - \nu) + 2\gamma r\big] \leq (2\gamma - \nu) - 2\nu r < (2\gamma - \nu) - \nu = 2(\gamma - \nu) < 0.$$

Thus $E$ is decreasing on this interval, and

$$E(u) \geq E(\bar{u}) = [(\nu - \gamma) + \nu r](\nu + \gamma)r > 0.$$

It follows that $N_L < 0$.

Eliminating one multiplier at a time from (13) gives

$$\widetilde{\mu}_S \mathcal{D} = \delta N_S, \qquad \widetilde{\mu}_L \mathcal{D} = \delta N_L.$$

Since $N_S > 0$, the first equation requires $\widetilde{\mu}_S > 0$ and $\mathcal{D} > 0$. The left side of the second equation is then nonnegative, whereas its right side is negative. This contradiction proves

$$Q_S > \gamma m_S, \qquad \text{and hence} \qquad \rho < 1.$$

The own- and cross-cost retail-price responses are the same as in symmetric Bertrand. It follows that

$$\rho^* - 1 = \frac{(2-\gamma)(q_L - q_S)}{\gamma q_L + 2q_S} \geq 0.$$

Therefore $\rho < 1 \leq \rho^*$, and Proposition 2 gives

$$\frac{d\text{CS}}{dm_L} < 0.$$

It remains to determine how $m_L$ changes with buyer power. The Jacobian of the two binding participation constraints with respect to $(m_S, m_L)$ is

$$J_{\text{asym}} = -\frac{2\nu}{\delta^2} \begin{pmatrix} Q_S & \gamma m_S \\ \gamma m_L & Q_L \end{pmatrix}.$$

We have shown that

$$Q_L \geq Q_S > \gamma m_S > \gamma m_L.$$

Consequently,

$$\det J_{\text{asym}} = \frac{4\nu^2}{\delta^4} \left( Q_L Q_S - \gamma^2 m_L m_S \right) > 0.$$

Because the two participation equations are continuously differentiable and $\det J_{\text{asym}} > 0$, the implicit-function theorem applies. Thus, for $\lambda$ near $1/n_L$, the two binding participation constraints uniquely determine $m_S(\lambda)$ and $m_L(\lambda)$, and both costs are continuously differentiable. Differentiating these equations gives

$$\frac{dm_L}{d\lambda} = \frac{F(2\nu Q_S/\delta^2)}{\det J_{\text{asym}}} > 0.$$

These derivatives compare nearby offers that keep both buyers indifferent between accepting and switching. Since $d\text{CS}/dm_L < 0$, it follows that $d\text{CS}/d\lambda < 0$. A marginal reduction in $\lambda$ therefore raises consumer surplus.

Thus any equilibrium at which a marginal reduction in $\lambda$ fails to raise consumer surplus must have $q_L < q_S$. ■

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