Market microstructure and informational complexity

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Abstract

Competitive markets feature minimal informational complexity; agents only need to know prices to implement an efficient allocation. However, the standard formulation of competitive equilibrium neglects the mechanism of price formation, treating prices as exogenous. We study two explicit price formation mechanisms: trade intermediated by market-makers and direct trade via random matching and bargaining. We show that as the economy grows, the informational complexity of the random matching diverges to infinity relative to the competitive market. This divergence can be avoided if market makers intermediate trade, providing a novel rationale for market-making if agents' capacity to deal with complexity is limited.

Keywords: allocation mechanisms, informational efficiency, random matching equilibrium, market-makers, intermediation.

JEL-Classification: C78, D40, D60, D82, D83.

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1 Introduction

Economists have long noted that competitive markets exhaust gains from trade; that is, they are Pareto efficient. This paper studies a second metric economists have used to argue that markets are desirable. Market prices communicate all the relevant information dispersed throughout the economy; that is, competitive markets minimize *informational complexity.* To use the famous example from Hayek (1945, p. 526), when the price of tin increases, "All that the users of tin need to know is that some of the tin they used to consume is now more profitably employed elsewhere." For the tin user, knowing that a shift in either supply or demand caused the price increase would be redundant information; the price provides all the information regarding market conditions that the user needs. Our paper argues that the specific mechanism of price formation matters for the market's informational complexity. We show that a formal market mechanism, articulated as an intermediating agent who sets prices, plays a fundamental role by providing the prices that communicate information to the agents; otherwise, if buyers and sellers have to "produce" the market price by themselves, then this result is an explosion of informational complexity.

Mount and Reiter (1974), Hurwicz (1977b), and Jordan (1982) showed that competitive markets, where agents take prices as given, feature "minimal information complexity."[1] However, as Gale (2000) and others have asked: "Who sets the prices in a competitive equilibrium?" They argue that instead of interpreting the model of competitive equilibrium as a literal description of the behavior of agents in the economy, it should be interpreted as a simplified representation of an economy where prices are strategically determined, the frictions of trade are low, and the number of agents is high. Building on this work on strategic foundations for competitive equilibrium, we argue that markets should be thought of as minimizing informational complexity only if markets where strategic agents explicitly set prices can approximate the informational complexity of the competitive equilibrium. Therefore, our paper studies informational complexity in markets *with strategic price-setters* and without the Walrasian auctioneer.

To study markets with strategic agents, we need to describe the market structure explicitly and

[1]The term they used was "informational efficiency," however, because this expression today is used in economics and finance to mean something else, we use the term "informational complexity." See Section 2 for further discussion.

model intermediaries and other institutions that facilitate trade Spulber (1996b, p. 135). We compare direct trade between randomly matched buyers and sellers, as in models such as Diamond (1982), Mortensen and Pissarides (1994), Kiyotaki and R. Wright (1989), Mortensen and R. Wright (2002), and Duffie, Garleanu, and Pedersen (2005), to indirect trade mediated through explicit market-makers/market institutions, as in models such as Gehrig (1993), Spulber (1996a), Spulber (2002), and Rust and Hall (2003).[2] We show that direct trade tends to explode in terms of informational complexity and that indirect trade mediated through intermediaries approximates the informational complexity of the competitive equilibrium.

In indirect trade mediated through explicit market-makers, buyers and sellers outsource the price formation process to intermediaries, with positive trade frictions, the market-makers profit by extracting part of the surplus from trade. These market-makers are arbitrageurs who can buy low, sell high, and exploit opportunities other actors do not see.[3] As trade frictions go to zero, the market-makers still economize on information for the other agents, but the arbitrage opportunities disappear, market-makers no longer make any profits, and the allocation converges to the competitive allocation.[4]

We then extend the general market-maker model to a dynamic setting with a monopoly market-maker. Other potential market-makers can pay a fixed cost to enter, and the market is "contestable" as in Baumol (1982). In equilibrium, the monopoly incumbent will deter entry and set all prices like the Walrasian auctioneer. Unlike the auctioneer, the market-maker is strategic, and prices are endogenous. The downside is that, with positive frictions, there are different bid and ask prices for the good instead of the single price from the auctioneer. The spread means that to implement the market-maker allocation, the message space requires *only* one more dimension than the unique minimally complex competitive market, making an economy with a monopoly market-maker second-best compared to the competitive market.

Given that the market-maker microstructure approximates the low information complexity of

[2] Our model has similarities with the intermediation models of Gehrig (1993), Spulber (1996a), Spulber (1996b), and Rust and Hall (2003). Following Rust and Hall (2003), we use the term market-maker because they operate an exchange. There are subtleties of the market microstructure that we do not study. For example, we do not address the difference between being a merchant or a platform Hagiu (2009) or a marketplace or a reseller Hagiu and J. Wright (2015). See Spulber (2019) for a recent discussion.

[3] Following Kirzner (1973, pp. 14-6) there is a literature that calls these market-makers "entrepreneurs" who "discover" profit opportunities in the market.

[4] The formal connection between no-arbitrage and competitive equilibrium is well understood in the case of product markets (e.g., Makowski and Ostroy 1998) and financial markets (e.g., Werner 1987).

the competitive market, it is natural to ask whether that is a feature of other microstructures. Our second result shows that the answer is no: we show that a standard, random matching model, as articulated in Mortensen and R. Wright (2002), is unattractive from an informational complexity perspective. If matching frictions are small, the random matching mechanism is approximately competitive and, therefore, efficient. However, matching remains inefficient from an informational complexity perspective. We show that the random matching and bargaining allocation mechanism requires infinitely more information than the competitive mechanism as the number of types of agents in an economy grows. Random matching is informationally inefficient because each participating agent needs to have an exhaustive picture of market conditions.

The paper is structured as follows: Section 2 discusses related literature. Section 3 lays out the abstract environment within which we will consider the specific mechanisms. Section 4 explains the baseline competitive mechanism. We then construct our market-maker in Section 5. Section 5.4 develops a dynamic model with a monopoly market-maker. Section 6 describes the matching and bargaining mechanism and our result on informational inefficiency. Section 7 concludes.

2 Related Literature on Informational Complexity and the Foundations of Competitive Equilibria

The groundbreaking work of Mount and Reiter (1974), Hurwicz (1977b), Hurwicz (1977c), and Jordan (1982) has shown that competitive markets require *minimal information.* Every agent can be unaware of most of the economy, and their preferences are private. Mount and Reiter (1974) showed that competitive equilibria are informationally efficient in the sense that competitive prices communicate the minimum amount of information necessary to implement a Pareto efficient allocation in an environment where information is dispersed. Jordan (1982) proved that competitive prices are the *unique* decentralized mechanism that achieves informational efficiency and satisfies the individual rationality constraint: Jovanovic (1982) showed that for any allocation mechanism that satisfies the individual rationality constraint (which he defined as "non-coercive"), implements a Pareto efficient allocation, and is informationally efficient, then that mechanism is the competitive allocation mechanism, that is, the mechanism implements the competitive equilibrium allocation. Thus, Mount and Reiter (1974) showed the analogous result to the first welfare theorem for informational complexity, and Jovanovic (1982) showed the analogous concept to the second welfare theorem for information efficiency.[5]

Informational complexity is a relevant metric to judge allocation mechanisms if decision-makers are constrained by the quantity of information they can incorporate into their decision-making process. In conventional economic theory, decision-makers maximize their utility regardless of the complexity of their decision problem. In contrast, if agents are boundedly rational Selten (2001) or have rational inattention Caplin and Dean (2015) and Maćkowiak, Matějka, and Wiederholt (2020), the amount of information matters. Recent work has explicitly incorporated these limitations when evaluating alternative mechanisms Li (2017) and Oprea (2020). Our paper suggests that certain allocation mechanisms are desirable institutions not because humans are perfectly rational instantaneous utility maximizers but because they are not. While we do not explicitly model any cognitive constraints, our results suggest organized markets function as a means to reduce cognitive costs.

Also, there is a large literature on strategic foundations for competitive equilibrium, summarized in Osbourne and Rubinstein (1990) and Gale (2000). Under various market microstructures, the strategic equilibrium of decentralized economies generates the same allocation as the competitive equilibrium.Our paper shows that minimal informational complexity is not a typical feature of those strategic mechanisms. In particular, the random matching and bargaining model is one popular explanation for economists to expect competitive allocations as frictions of trade are low (Gale 1986a; Gale 1986b), but a random matching and bargaining equilibrium requires a much greater quantity of information than a competitive equilibrium. Thus, matching markets can implement allocations that are approximately competitive when frictions are low, but they cannot describe the low informational complexity that Hayek 1945 understood to be a feature of markets.

[5]Sato (1981) and Tian (2004) extended informational complexity results to convex economies with public goods and externalities. More recently, Nisan and Segal (2006) extended this literature to non-convex economies and the analysis of the allocation of indivisible goods.

3 Environment

In this section, we define our physical environment and abstract allocation mechanisms. In the next sections, we describe three specific allocation mechanisms and their properties in terms of informational complexity: the competitive market in Section 4, the market-maker mechanism in Section 5, and the random matching mechanism in Section 6.

3.0.1 Allocation mechanisms

Consider an economy with N individuals. For each individual $I \in 1, 2, \ldots, N$, let E^i be the set of "individual environments," which specifies endowments and preferences for each individual. Then, the set of possible environments E is the product of the set of individual environments, so $E = \prod_i E^i$.

We let M be an abstract message space and Y be the set of feasible net trades for the individuals of this economy. The non-empty valued correspondence $\mu : E \rightrightarrows M$ specifies a set of messages for each environment. Finally, the outcome function $g : M \to Y$ then maps messages to net trades.

Putting this together, we can define an **allocation mechanism**, following Mount and Reiter (1974), Hurwicz (1977b), and Hurwicz (1977c), and Jordan (1982):

Definition 1. An *allocation mechanism* is a triple (μ, M, g).

We call (μ, M) the **message process** of the allocation mechanism (μ, M, g): the message process is the correspondence that specifies messages given each environment and the message space M.

We are interested in informationally decentralized allocation mechanisms, which are mechanisms that feature a message process (μ, M) that is **privacy-preserving**:

Definition 2. A message process (μ, M) is *privacy-preserving* if for each i there exists a correspondence $\mu^i : E^i \rightrightarrows M$ such that for each $e = (e^1, e^2, \ldots, e^N) \in E$, the profile of correspondences $(\mu^i)_{i\in\{1,\ldots,N\}})$ satisfies

$$\mu(e) = \cap_{i\in\{1,\ldots,N\}}\mu^i(e^i).$$

In other words, a mechanism is **privacy-preserving** if each individual's response to a message only incorporates that person's information and not the information of others: this a desirable feature of a mechanism since only a consumer knows her endowment and preferences.

3.0.2 Physical environment

For simplicity, we study environments E where there are two goods: a consumption good and a numeraire good. As agents can consume only positive quantities of the consumption good, the consumption set is $X = \mathbb{R}_+ \times \mathbb{R}$. There is a continuum of consumers in this economy of measure one. There are N types of consumers in this economy, each of identical measure $1/N$. A type $i \in \{1, \ldots, N\}$ has preferences defined on X by a quasilinear utility function u_i that satisfies for $x = (x_1, x_2) \in X$, that $u_i(x) = u_{1i}(x_1) + x_2$, and u_{1i} is strictly increasing, concave, and continuous. Let $\mathcal{F}$ be the set of such functions and let $w_i \in X$ be the endowment of consumers of type i.

A specific environment is a realization of $e \in E$ that specifies a profile of quasilinear preferences for the types $(u_i)_{i\in\{1,\ldots,N\}} \in \mathcal{F}^N$ and a profile of endowments for each type $(w(i))_{i\in\{1,\ldots,N\}} \in X^N$. Thus, $e = (u_i, w(i))_{i\in\{1,\ldots,N\}}$ and $E = \mathcal{F}^N \times X^N$.

Let $y \in \mathcal{R}^{2N}$ be a vector of net trades for all individuals. Let Y be the set of feasible net trades which satisfies

$$Y = \{y = (y_i)_{i=\{1,\ldots,N\}} : \sum_i y_i = 0, y_i + w_i \in X \quad \forall i\}.$$

In addition, an allocation mechanism (μ, M, g) is said to be **non-coercive** (that is, satisfies the participation constraint) if the allocation implemented by the mechanism yields a higher utility than consuming the endowments. This definition is stated formally below:

Definition 3. The mechanism (μ, M, g) is non-coercive if for each $y \in g(\mu(e))$ then $u(w_i + y_i) \geq u(w_i)$ for all $i \in \{1, \ldots, N\}$.

4 The Competitive Allocation Mechanism

The competitive mechanism is a triple (μ_c, M_c, g_c). The message space M_c is described by

$$M_c = \{(p, y) \in \mathcal{R}^2_{++} \times Y : py(i) = 0 \ \ \forall i\}. \tag{1}$$

In words, it is the set of prices and net trades that preserve the budget balance of all consumers.

Consider a message correspondence μ_c^i for each consumer i:

$$\mu_c^i(e^i) = \{(p, y) \in M_c : y(i) \in \arg \max_{y \in \{z \in \mathbf{R}^2 : pz = 0\}} u_i(y)\}. \tag{2}$$

The message correspondence for the competitive allocation mechanism μ_c is the intersection of these correspondences:

$$\mu_c(e) = \cap_i \mu_c^i(e^i). \tag{3}$$

In words, the competitive message correspondence consists of prices and allocations that map the set of physical environments into messages that maximize utility for each consumer. By construction, the message correspondence is privacy preserving.

The outcome function g_c just maps the message space into the set of net trades:

$$g_c((p, y)) = y. \tag{4}$$

Thus, the reader can check that for an environment $e \in E$, $\mu_c(e)$ yields the competitive equilibrium allocation and prices: it specifies prices and net trades that maximize the utility of consumers and are feasible, and the set of "outcomes" $g_c(\mu_c(e))$ describes the set of equilibrium net trades for the environment e. Thus, if the competitive equilibrium exists for an environment $e \in E$, then $\mu_c(e)$ and $g_c(\mu_c(e))$ are non-empty.

In the competitive mechanism of the N-types economy, the message space includes only one price (as the price of the numeraire good is normalized to 1) and N types of consumers minus one for market clearing. Therefore, we have the following lemma:

Lemma 1. *The message space of the competitive mechanism M_c is N-dimensional in the* *sense that it is diffeomorphic to an N-dimensional manifold.*

Proof. See Appendix Subsection B.1 □

Jordan (1982) showed that, under mild regularity conditions, the competitive mechanism is the unique noncoercive mechanism that achieves minimal informational complexity in implementing a Pareto efficient allocation. That is, any other non-coercive mechanism that implements a Pareto efficient allocation requires a higher dimensional message space. Thus the competitive mechanism is the benchmark for other mechanisms.

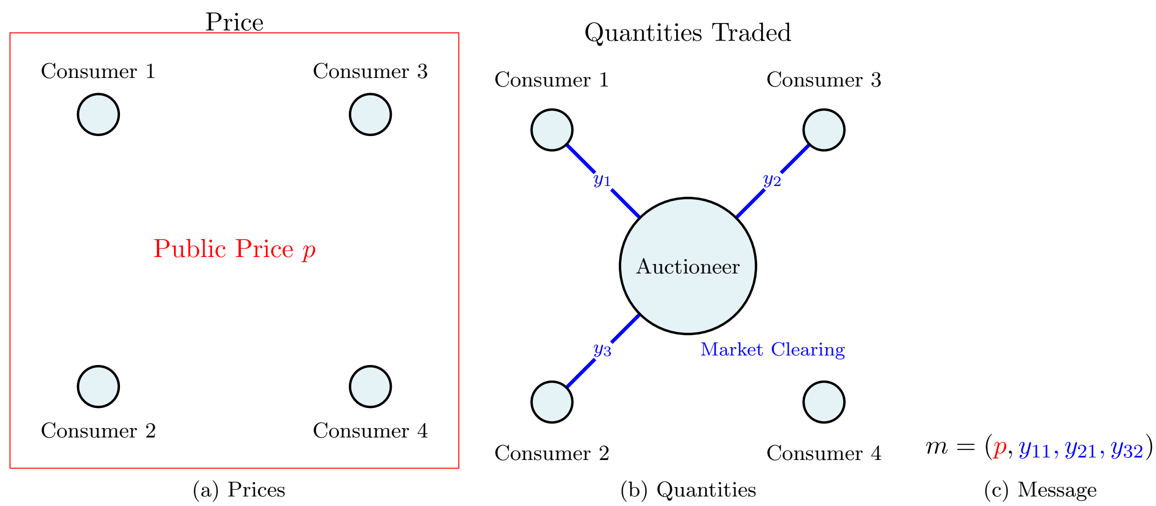


Figure 1: Example Competitive Mechanism with $N = 4$

For example, consider an economy with four types in Figure 1. The allocation mechanism needs to communicate the relevant information about both prices and quantities. There is one public price for the consumption good for all consumers to know, as shown in Figure 1a; the price of the numeraire good can be normalized to one. For the quantities, the mechanism informs each consumer of the quantity of the consumption good to trade; the quantity of the numeraire is implicitly given by the budget balance. Also, if we know the net trades for all but one type, then market-clearing implies the net trades for the last type. Therefore, the dimensionality of the message space is 4: one price and net trades of the first three types of consumers.

5 Informational Complexity under the Market-maker Mechanism

Instead of a perfectly competitive market, in this section, we consider explicit market microstructure: there is a finite set J of "market-makers" in this economy. Market-makers are profit-maximizing[6] intermediaries that "make the market" by posting bid and ask prices for the consumption good and intermediate trade between the consumers.[7] Buyers purchase from the lowest-priced market-maker they have access to as long as it is lower than their valuation, while sellers sell at the highest-priced market-maker as long as the posted price is higher than their cost. For simplicity, in this section, we suppose that the utility function of the consumers for the consumption good is strictly concave, so the competitive equilibrium is unique.[8]

5.1 Frictionless Environment

First, consider the case where consumers have full and costless access to the prices posted by all market-makers. This environment implements the competitive mechanism through Bertrand competition. To see this, consider a market-maker who posts a pair of bid and ask prices $(p_b, p_s) \in \mathbb{R}^2_{++}$ for the consumption good, which are, respectively, higher and lower than the prices posted by all other market-makers. In that case a consumer will either purchase the consumer good for p_s or sell the consumer good for p_b.

Let $D_i(p)$ be the demand of a consumer of type i for the consumption good. As there are bid and ask prices, consumers partition themselves into two groups: the types that have excess demand for the good and choose their demand according to $D_i(p_s)$ and the types who have excess supply who choose the quantity according to $D_i(p_b)$. Because demand is downward sloping, which means here $D_i(p_s) < D_i(p_b)$ if $p_b < p_s$, there might be some types i where $D_i(p_s) \leq w_{i1} \leq D_i(p_b)$: some people may be net suppliers, and some may be net demanders.

[6] Only attribute utility to the numeraire good.

[7] In our model, market-makers perform the same role as in Spulber (1996a) and Spulber (1996b). We have a finite number of market-makers, and consumers are matched with different probabilities to each market-maker.

[8] As the utility function is strictly increasing, demand is single-valued, continuous, and strictly decreasing on price, implying a unique competitive equilibrium price p^*.

The market-maker chooses (p_b, p_s) to maximize profits, which are

$$\pi(p_s, p_b) = (p_s - p_b) \times \left[\sum_{i:D_i(p_b)<w_{i1}} [w_{i1} - D_i(p_b)] \right], \tag{5}$$

subject to the market clearing constraint that the quantity brought from sellers is equal to the quantity demanded by buyers:

$$\sum_{i:D_i(p_b)<w_{i1}} w_{i1} - D_i(p_b) + \sum_{i:D_i(p_s)>w_{i1}} w_{i1} - D_i(p_s) = 0. \tag{6}$$

If the market-maker posts bid prices lower than some other market-maker, then no seller will sell to it, and its profits will be zero. If the maker posts bid prices higher than all others but not the lowest ask prices, the market-maker has monopolized the supply, and profits also satisfy Equation 5 subject to the resource constraint 6.

Proposition 1. *If at least two market-makers are operating, then there is only one Nash equilibrium: for at least two market-makers to post a pair of bid-ask prices $(p_b, p_s) = (p^*, p^*)$; market-makers post the competitive equilibrium price.*[9]

Proof. See Appendix Subsection B.2 □

5.2 Frictions of Trading: Constrained Consideration Sets

The simple strategic model of intermediation studied in the previous section can provide strategic foundations for competitive markets, but as real markets exhibit many imperfections, here we construct an extension of this model that allows its equilibrium to feature "imperfections" of markets such as market power and price dispersion.

Suppose that consumers have constrained access regarding the market-makers that they can trade with.[10] For each market-maker $j \in J$, let $m^j \in (0, 1]$ be the fraction of consumers with

[9] Note that the market clearing constraint prevents a market-maker from monopolizing the supply and posting the monopoly price as an equilibrium: that would be part of an equilibrium if the demand from the buyers were inelastic. In that case, a market-maker could make zero profits by posting a higher ask price than the competitive price and selling part of the supply at the (revenue maximizing) monopoly price. However, the quantity supplied at a higher ask price is strictly higher, and the quantity demanded is strictly lower than in competitive equilibrium. Thus, there will be excess supply, and the market-maker will not satisfy the resource constraint (as we do not allow excess supply in our market-clearing condition). This explains why our set of possible equilibrium is restricted compared to Stahl (1988).

[10] Other studies, such as Perla (2019), Guthmann (2024), McAfee (1994), Brian C. Albrecht (2020), and Arm-

access to market-maker j. We assume that access is randomly and independently distributed, so a fraction m^j of consumers of any type has access to market-maker j and the fraction of consumers with access to market-makers j and j' is $m^j \times m^{j'}$. Independence also implies that the fraction of consumers who are aware of the seller j conditional on being aware of a competitor is m^j. A consumer's access to market-makers is private information, so market-makers cannot discriminate consumers based on their access.

A consumer type is defined by its utility function u_i and accessibility $A^i \subset J$. As access is independently distributed, if $m^j < 1, \forall j$, then there is a positive measure of consumers without access to any market-maker. Let NA (for "no-access") be the index for this set of consumers A^{NA}. The budget set for a consumer of type i includes all pairs of prices from market-makers that they have access to:

$$B_i = \{y \in X - w_i : \exists j \in A^i \text{ such that } p^j(y)y = 0\}, \tag{7}$$

where $p^j(y)$ are the prices posted by market-maker j conditional on net-trade y (that is, if the consumer chooses a positive quantity of the consumption good, the price is p^b, and if the consumer chooses a negative quantity, the price is p^s).

As consumers' valuations and access to market-makers are private information, market-makers are constrained to uniform pricing policies where there is no price discrimination. In this case, each market-maker posts a pair of bid and ask prices, and the consumers choose the best prices among the market-makers they can access.

We describe a strategy of the market-makers by a pricing function p that assigns bid and ask prices for an $\alpha \in [0,1]$, which represents the fraction of the monopoly profit that can be extracted from the consumers. Let $\mathrm{p}(\alpha) = (p_b(\alpha), p_s(\alpha))$ and note that $(p_b(1), p_s(1)) = (p_b^M, p_s^M)$, the monopoly price that maximizes a market-maker's profits conditional on it being a monopoly (note they satisfy market-clearing to be feasible). The profits earned from the consumers from trades that are executed when there is a pair of bid and ask prices $\mathrm{p}(\alpha)$ are a fraction α of the monopoly profits, which we denote by Π^M.[11]

strong and Vickers (2022), use terms such as "awareness," "availability rate," "choice set," "loyal customers," and "consideration set" to indicate the subset of agents that buyers or sellers have access to.

[11] Access is independently and uniformly distributed. Therefore, the quantities bought and sold by consumers in response to a pair of bid and ask prices are proportional to the access parameter m^j, which means that in an

A profile of actions is described by $\{\alpha^j\}_{j\in J}$ for each market-maker. A mixed strategy profile is a profile of cumulative distribution functions $\{P^j\}_{j=1}^J$ on $[0,1]$ that for $\alpha \in [0,1]$ assigns a cumulative probability $P^j(\alpha) \in [0,1]$ of posting bid and ask prices that yield lower profits than $(p_b(\alpha), p_s(\alpha))$.

An equilibrium is a profile of mixed strategies $\{P^j\}_{j=1}^J$ such that posting a pair of bid and ask prices p(α) for α on the support of the distribution P^j is profit-maximizing for market-maker j. As stated in Proposition 2, given a profile of access parameters $\boldsymbol{m} = (m^j)_{j\in J}$ such that there exists at most one market-maker of whom all consumers are aware of, there is a unique equilibrium strategy profile $\{P^j\}_{j=1}^J$ in this environment. The equilibrium mixed strategy profile described in Proposition 2 has the following properties: the distributions of prices posted by the market-makers are non-degenerate and are continuous on the interior of the support, and the larger market-makers (in terms of the m^j) transact at higher profit margins than smaller market-makers in the sense that the distribution of margins between ask and bid prices of the larger market-makers first-order stochastically dominate those of the smaller market-makers. The reason for this result is that (since access is uniformly and independently distributed) it is less likely that buyers and sellers have access to a competitor of a large market-maker than a competitor of a smaller market-maker, so the larger market-maker loses fewer customers if the spread between the buy and sell prices is increased.

Proposition 2 (Market-maker Equilibrium)**.** *If $\boldsymbol{m}$ is such that $m^j < 1$ for at least $J-1$ market-makers, then there is a unique equilibrium that consists of a profile of mixed pricing strategies $\{P^j\}_{j\in J}$ and a sharing rule: for a pair market-makers h and g, if $m^h < m^g$, then consumers with access to both will trade with h if the posted prices are the same.*

The profile of equilibrium strategies defined on $[0,1]$ features connected supports $[\underline{\alpha}^j, \overline{\alpha}^j]$ for each market-maker $j \in J$, which share a common lower bound of the support $\underline{\alpha}$. The distributions are continuous on $[\underline{\alpha}, 1)$. For each $j \in J$, for $\alpha \in [\underline{\alpha}^j, \overline{\alpha}^j]$, $P^j(\alpha)$ satisfies

$$P^j(\alpha) = \frac{m^{\bar{j}}}{m^j} P^{\bar{j}}(\alpha),$$

economy with one market-maker who posts p(α), then the profits created by that market-maker are a fraction α of the profits under monopoly prices. In addition, independence implies that for an economy populated by two market-makers j and j', if j is playing prices according to α^j and j' is playing $\alpha^{j'}$ with $\alpha^{j'} < \alpha^j$—which implies consumers prefer the prices by j'—then the profits of j are $m^j(1-m^{j'}) \times \alpha \times \Pi^M$. Independence also implies that if the prices posted by j imply feasible net trades under monopoly (so the quantity supplied equals the quantity demanded), then if j loses consumers to the competition of j's proposed prices, that implies a proportional loss of quantity supplied and demanded (in both cases equal to $m^{j'} \in (0,1]$), which means that feasibility still holds.

where $\bar{j}$ is the market-maker with the largest awareness parameter m. The distribution $P^{\bar{j}}$ is given by

$$\prod_{j \neq \bar{j}} (1 - m^j P^{\bar{j}}(\alpha))\alpha = \prod_{j \neq \bar{j}} (1 - m^j).$$

Proof. See Appendix Subsection B.3. $\square$

The reader can note that the equilibrium strategies converge to the competitive equilibrium as consumers approach full access to at least two market-makers:

Corollary 1 (Convergence to Competitive Equilibrium)**.** *Consider a sequence of access parameter profiles $\boldsymbol{m}_n$ for the market-makers. If, for at least two market-makers h, g, m_n^h and m_n^g both converge to one, then the equilibrium pricing strategies $\{P^j, \boldsymbol{p}\}_{j \in J}$ converge in probability to a competitive equilibrium price p^*.*

5.3 Allocation Mechanism

The allocation mechanism implicit in the market maker game, in this case, implements the allocation corresponding to a realization of the Nash equilibrium in mixed strategies. Note that if there is imperfect access regarding almost all the market-makers (that is if $(m^j)_{j=1}^J$ satisfies $m^j = 1$ for at most one j), then for any market-maker the posted price for buying is strictly smaller than for selling with probability one, and therefore profits are strictly positive. Following Hurwicz (1977a), we can interpret the profits of the market-makers and the resulting deadweight losses to be both components of the "cost" of operating the allocation mechanism.

The set of net trades incorporates the possibility of market-makers making profits by buying at lower prices than they sell. Let Y_m be the set of net-trades in this allocation mechanism, which are defined for each market-maker and each consumer type. Let Y_j be the set of net-trades for market-maker j, it is described by

$$Y_j = \{(y_i^j)_{\{i=1,\ldots,N\}} \in \mathbb{R}^{2N} : \sum_i (y_{i1}^j, y_{i2}^j) \in (0, \mathbb{R}_-)\}. \tag{8}$$

Note that if $j \in J$, then the set of trade trades with j must be feasible (so the quantity of the consumption good sold and brought must add up to zero).

As some consumers do not have access to any market-maker, let Y_{NA} correspond to the net trades implemented by the mechanism to consumers without access to any market-maker, where consumers cannot trade. In this case, only the null-set is an element of the set of net-trades:

$$Y_{NA} = \{(y_i^{NA})_{\{i=1,\dots,N\}} = (0,0)^N\}. \tag{9}$$

Then, the set of net trades Y_m is described by

$$Y_m = \{\{y_j\}_{j \in J \cup \{NA\}} : y_j = (y_i^j)_{\{i=1,\dots,N\}} \in Y_j\}$$

where j is either a market-maker (so $j \in J$) or $j = NA$, which indicates that a market-maker is not available.

Given a realized profile of prices $p_m = (p_s^j, p_b^j)_{j \in J}$, the message space is given by

$$M_m = \{(p_m, y) \in \mathbb{R}^{2J}_{++} \times Y_m : \forall j \in J, p^j y_{i1} + y_{i2} = 0\}.$$

To construct the privacy-preserving message correspondence we define the correspondences for each consumer type i, where $\mu_m^i : E^i \rightrightarrows M_m$ satisfies for any $(p_m, y) \in \mu_m^i(e^i)$ that the vector of net-trades for consumer i, (y_i1, y_i2), is utility maximizing given the profile of bid and ask prices that consumer of type i has access to among market-makers in A^i, that is $(y_i1, y_i2) \in \arg\max_{y \in B_i} u_i(w_i + y)$. Then, μ_m is a correspondence on E to M_m that satisfies

$$\mu_m = \cap_i \mu_m^i(e^i).$$

5.3.1 Informational Complexity of the Allocation Mechanism

In this environment, the dimensional size of the message space incorporates the different market-makers that make the market: if there are k market-makers, then there are $2k$ different prices posted to the consumers, and there is a subset of consumers who lack access to any market-maker. The cardinality of the set of consumer types is $N(k! + 1)$ as consumers can have access to either any subset of the market-makers or none. However, consumers only trade with the market-maker in his accessibility set with the most favorable prices. Therefore, we can represent the set of consumer types in the allocation mechanism by a coarser set of consumer types that only describes his utility function and the market-maker that he trades with.[12] Thus, the set of types has cardinality $N \times k + 1$, if a positive fraction of consumers lack access to any market-maker, since all types of consumers that do not have access to a market-maker have null net trades, and $N \times k$ if all consumers have access to at least one market-maker.

Given a realization of prices of the equilibrium price-posting game among market-makers, market-clearing among consumers who interact with each market-maker implies that the message space corresponding to environments with N different utility functions for consumers is Z-dimensional, where Z is equal to $2k + (N-1)k$ if all consumers have access to a market-maker or $2k + (N-1)k + 1$ if not. This is so because there are bid and ask prices for the consumption good posted by each market-maker (thus $2k$ prices), and net trades are defined for either Nk or $Nk + 1$ types of consumers, and the net-trade implemented for each type of consumer can be represented in one dimension as we know the price.[13] This implies the following proposition:

Proposition 3. *As the number of types of preferences N increases to infinity, the ratio of the dimensional size of the message spaces of the market-maker mechanism to the competitive mechanism converges to k.*

That is, the ratio of the size of the message spaces between the competitive mechanism and the market-maker mechanism is approximately the number of market-makers operating in the market. This result is intuitive since the competitive mechanism implicitly assumes a single monopolist market-maker called the Walrasian auctioneer whose bid and ask prices have zero spread.

[12] That is, the computation of the dimensional size of the message-space does not need to include the information of which other market-makers the trader was aware aside from the one she transacted with.

[13] The net trade can be represented as a quantity of the consumer good and the quantity paid/received by the consumer of the numeraire is implicitly implied by the budget constraint. Formally, it means we can construct a C^{∞}- diffeomorphism between the message space and a Euclidean space of either $2k + (N-1)k$ or $2k + (N-1)k + 1$ dimensions.

5.4 Contestable Markets

Our model of market-makers has a message space that is approximately proportional to the number of market-makers. Thus, this model implies that implementing an approximation of the competitive allocation requires at least twice the amount of information (an economy with two market-makers). Here, consider an extension of the model into a dynamic environment with contestable market-making, which allows the economy to approximate the informational complexity of the competitive mechanism by considering a monopolist market-maker who can deter the entry of other market-makers. If all consumers have access to the monopolist, the number of consumer types is N, but a pair of prices is realized instead of one price in the case of the competitive mechanism. This case represents the most minimally complex allocation mechanism in this class of market-maker environments, with informational size $N + 1$: one dimension more than the competitive mechanism. This additional dimension reflects the profit margin between purchase and sale to incentivize the market-maker to "produce" the price mechanism.

In this environment, time is discrete, $t = 0, 1, 2, \ldots$, and let $\beta = 1/(1 + r)$ be the discount factor. The consumption good is perishable, and consumers' endowments can be interpreted as a constant stream of the perishable consumption good.

Accessibility Diffusion: Given a set J of market-makers, there is an accessibility profile $\{m_t^j\}_{j=1}^J \in (0, 1]^J$. Suppose accessibility regarding a market-maker diffuses through the economy according to

$$m_{t+1}^j = (1 - \delta)m_t^j + M(m_t^j, 1 - m_t^j), \tag{10}$$

where M (following Guthmann 2023) is a matching function that represents the diffusion of accessibility through consumers who hitherto had access to the market-maker, and $\delta \in [0, 1)$ is the rate at which consumers lose access to a market-maker (i.e., accessibility depreciation parameter).

In period zero, each market-maker chooses to post prices according to a sequence of distributions for each period. Since the choice of the pricing strategies does not have any effect on the state of the market, the optimal strategy for each market-maker is to choose the profit-maximizing pricing behavior in each period, given the action profile of the other market-makers in that period. Therefore, at time t the prices practiced in the market are $\{P^j\}_j$, described in Proposition 2 with accessibility profile $\{m^j\}_j$.

We are interested in the convergence of equilibrium to the competitive equilibrium: let D be the probability that the prices consumers have access to are posted from a distance $\epsilon > 0$ from competitive equilibrium prices. The economy converges to the competitive equilibrium when D converges to zero. Proposition 4 follows from Proposition 2, as the expected equilibrium margin between bid and ask prices posted by the market-makers converges to zero if $\lim m_t^j = 1$ for $m_t^j > 0$ and $J \geq 2$. Therefore, consumers' accessibility regarding the market-makers operating converges to one as $t \to \infty$. This implies that consumers are converging to having access to bid and ask prices that are converging in probability to the competitive price.

Proposition 4. *If there are at least two market-makers and if the law of motion for accessibility diffusion (equation 10) implies that* $\lim m_t^j = 1$ *for* $m_t^j > 0$*, then as* $t \to \infty$ *the equilibrium prices and the equilibrium allocation converge in probability to the competitive equilibrium.*

5.4.1 Contestable Market Equilibrium

We will now show how a single market-maker ($N = 1$) approximates the competitive outcome if the market is contestable in the sense of Baumol (1982). To model a contestable market, we introduce introduce entry. To represent the possibility of entry, let a potential entrant be a market-maker j with accessibility parameter $m_t^j = 0$. This potential entrant can enter the market incurring an entry cost $E > 0$, which is the cost of setting up an entry-level accessibility parameter $m_e \in (0, 1)$.

There are two market-makers indexed by 1 and 2. Further, at a date, $t = 0$ suppose that $m_0^1 = 1, m_0^2 = 0$, that is, in period 0, market-maker 1 is a monopolist that all consumers have access to and market-maker 2 is out of the market. However, 2 can decide to enter at any period. A monopoly deterrence equilibrium is a situation where the incumbent market-maker 1 posts a pair of bid and ask prices in each period such that the profits of a prospective market-maker from offering better prices to consumers are too low to compensate for the cost of entering the market.

Definition 4. A *monopoly deterrence equilibrium* is an equilibrium where 1 chooses a pricing schedule and, given this pricing schedule, 2 finds it optimal not to enter. The pricing schedule is profit-maximizing for two reasons. First, a higher bid-ask margin that yields higher profits for 1 would mean that 2 would enter and undercut 1's posted offers in every period. Second, the schedule is profit-maximizing in the sense that it yields a higher discounted expected value of the profit stream for 1 than the expected value of the profits in the equilibrium under a duopoly if 2 also enters the market.

The proposition below states that if entry costs are high enough and accessibility diffusion is fast enough, then the unique equilibrium is for the monopolist to deter entry. This occurs as the entry cost is higher than the expected profits that can be obtained in the duopoly competition. However, monopolist 1 must commit to a sequence of prices that still deters the entrant. The unique equilibrium is the sequence of prices that makes 2 indifferent between entering and not entering, but that maximizes the present value of 1's profit stream. As the discount rate decreases, the present value of the gains from entering the market increases. This implies that the bid and ask prices posted by the monopolist become closer to the competitive equilibrium price. Therefore, even with a single active market-maker, when the discount rate is sufficiently low, competition "for the field"—to borrow a phrase from Demsetz (1968)—is sufficiently intense such that the equilibrium approximates the competitive equilibrium.

Proposition 5. *If accessibility diffusion is fast enough, such that $\sum_{t=0}^{\infty}(1-m_t^2) \leq C$ for some constant C conditional on market-maker 2's entry, and the discount rate r is low enough, then for an entry cost E equal or higher than $C \times \pi^M$, the unique equilibrium is the monopoly deterrence: The monopolist commits to post prices $\mathbf{p}(\pi)$ that yield a per-period profit of*

$$\pi = E/\left(\sum_{t=0}^{\infty} \beta^t m_t^2\right)$$

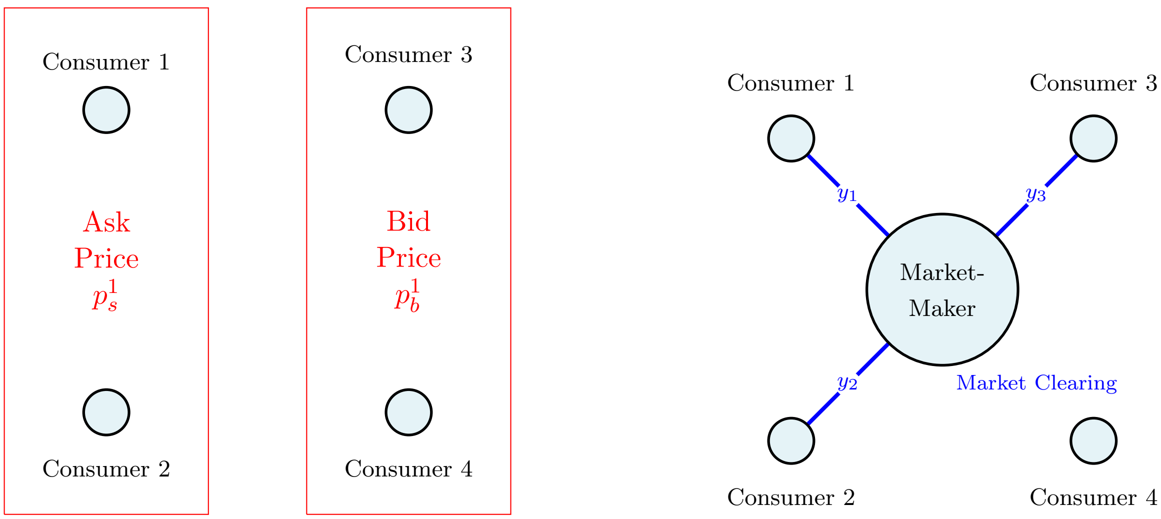
to deter entry. As r converges to zero, the deterrence monopoly equilibrium profit flow π converges to zero, which means the posted buying and selling prices converge to the competitive equilibrium prices p^*.

Proof. See Appendix Subsection B.4. □

Consider an example with two types of buyers and two types of sellers; the mechanism needs to communicate two prices, a price for buyers and one for sellers, as shown in Figure 2a. The communication for the quantities is the same as the competitive mechanism, except now the trades "go through" the market-maker and not the Walrasian auctioneer, as shown in

Figure 2b. The message space M_m is five-dimensional: two prices and three quantities. More generally, as stated in Proposition 6, our analysis implies that the dimensional size of the message space of the allocation mechanism is $N + 1$, which is only one dimension more than the competitive mechanism. This occurs because the market-maker posts a pair of prices instead of only a single price.

Proposition 6. *The message space of a monopoly deterrence equilibrium requires one more dimension than the competitive equilibrium.*



(a) Prices (b) Quantities traded

$$m = \left(p_b^1, p_s^1, y_1, y_2, y_3\right)$$

(c) Message

Figure 2: Example of the allocation mechanism of a monopolist market-maker with $N = 4$

6 Information Size without Intermediation

The competitive mechanism is meant to represent the frictionless limit of strategic price formation. We showed how the price formation mechanisms based on intermediation (where consumers trade through market-makers) can approximate minimal complexity. In this section, we consider the situation where consumers have to meet each other and bargain over the terms of trade. While these models of random matching and bargaining can arrive at the competitive allocation as frictions of trade decrease, we will see that they cannot approximate the minimal informational complexity of the competitive mechanism. To understand the complexity of a decentralized matching and bargaining process, we use the model from Mortensen and R. Wright (2002), which is a standard articulation of the modern random matching and bargaining modeling framework.

6.1 Description of the Matching Environment

Time is continuous, and for tractability, we assume that consumers are either buyers who are not endowed with the consumption good or sellers who are endowed with one unit of the consumption good. There are $N_b > 0$ types of buyers and $N_s = N - N_b > 0$ types of sellers, buyer types i_b are indexed by $i_b \in \{1 \ldots, N_b\}$ and seller types i_s are indexed by $i_s \in \{N_b + 1, \ldots, N\}$. Consumers have unit demand for the consumption good.[14] Let F and G be the cumulative distribution functions of valuations of buyers and sellers (the c.d.f. $F(x)$ is equal to the fraction of buyer types $\{i\}_i \in \{1, \ldots, N_b\}$ such that $v_i \leq x$).

There is a flow of buyers who can enter the market at the rate $b > 0$ and sellers at the rate $s > 0$. Buyers (sellers) then can choose to "enter the market," which means they can randomly meet sellers (buyers) and trade. Given populations of buyers $\mathcal{B} > 0$ and sellers $\mathcal{S} > 0$ participating in the market, they meet according to the matching function $M(\mathcal{B}, \mathcal{S})$. Let the buyer/seller ratio $\theta = \mathcal{B}/\mathcal{S}$ be the market tightness parameter, $m(\theta) = M(\mathcal{B}, \mathcal{S})/S$ be the rate a seller meets buyers, and $m(\theta)/\theta$ be the rate a buyer meets sellers. All agents discount future payoffs at the rate $r \geq 0$, and to participate in the matching process, buyers have to incur a cost $c_b \geq 0$, while sellers have to incur a cost $c_s \geq 0$.

When a buyer and a seller meet, one of the two, randomly chosen, announces a take-it-or-leave-it price offer. Let $\omega \in (0, 1)$ be the probability a seller makes the offer. If the other party rejects the offer, they both continue searching; if the other party accepts the offer, the exchange occurs, and both exit the market. We study the steady-state search equilibrium where the flows of buyers and sellers exiting the market are equal to the flows entering so that the corresponding net trades in the competitive equilibrium have an analogous implementation in this environment. That means b times the probability the buyers choose to enter is the

[14] That means a consumer of type i has a utility function $u_i(x_1, x_2)$ defined for the consumption good x_1 and the numeraire good x_2. The utility function satisfies $u_i(x_1, x_2) = v_i x_1 + x_2$ for $x_1 \in [0, 1]$ and $u_i(x_1, x_2) = v_i + x_2$ for $x_1 > 1$, we call v_i the valuation of type i.

flow of entering buyers in the market, and this flow of entering buyers is equal to the flow of matchings $M(\mathcal{B}, \mathcal{S})$ times the probability they trade and exit. In the steady-state equilibrium, the flow of sellers entering the market (supply) is also equal to the flow of trades and equal to the flow of buyers entering the market(demand).

The best strategy for one party is to offer the other party's reservation value; thus, the seller offers $p = x - V_b(x)$, the buyer offers $p = z + V_s(z)$, and a transaction occurs if and only if $x - V_b(x) \geq z + V_s(z)$. Since the seller makes the offer with probability ω, the expected price of a transaction is $p(x, z) = \omega(x - V_b(x)) + (1 - \omega)(z + V_s(z))$, which can be rearranged as

$$p(x, z) = z + V_s(z) + \omega[x - z - V_b(x) - V_s(z)]. \tag{11}$$

This is the price according to the generalized Nash solution if the seller captures a fraction ω of the joint surplus given reservation values $z + V_s(z)$ for the seller and $x - V_b(x)$ for the buyer.

Given the transaction prices, the values of participating in the market can be described as follows: the expected value of participation in the market for a buyer satisfies

$$rV_b(x) = \frac{m(\theta)}{\theta} \int \max\{x - p(x, z) - V_b(x), 0\} d\Gamma(z) - c_b, \tag{12}$$

and the expected value of participation in the market for a seller satisfies

$$rV_s(z) = m(\theta) \int \max\{p(x, z) - z - V_s(z), 0\} d\Phi(x) - c_s, \tag{13}$$

where Γ and Φ are the distributions of seller and buyer types participating in the market. These distributions differ from the respective exogenous distribution of potential seller and buyer entrants, G and F, as some types choose not to enter if the expected value of entering is not positive.

Substituting the right-hand side of equation 11 into equations 12 and 13 yields

$$rV_b(x) + c_b = \frac{m(\theta)(1 - \omega)}{\theta} \int \max\{x - z - V_b(x) - V_s(z), 0\} d\Gamma(z) \tag{14}$$

and

$$rV_s(z) + c_s = m(\theta)\omega \int \max\{x - z - V_b(x) - V_s(z), 0\} d\Phi(x). \tag{15}$$

Equations 14 and 15 show that the value of participating in the market is strictly increasing in the buyer's valuations and strictly decreasing in the seller's valuation. The steady-state search equilibrium is defined in terms of a pair of marginal types of buyers and sellers (R_b, R_s) with $R_b > R_s$, where a buyer with valuation x enters the market if and only if $x > R_b$, and the seller with valuation z enters if and only if $z < R_s$. In a steady-state search equilibrium, the distribution of types participating in the market is stationary, which implies that: (1) The measure of entering sellers and buyers must be the same, and therefore the pair of marginal valuations (R_b, R_s) satisfies the condition $sG(R_s) = b[1 - F(R_b)]$.[15] (2) The distribution of participating types is constant.

The steady-state search equilibrium is characterized by $(V_b, V_s, R_b, R_s, \Phi, \Gamma)$, the value functions (V_b, V_s), cutoff valuations to participate in the market (R_b, R_s), and the distributions of participating types (Φ, Γ) of buyers and sellers, respectively. The Appendix section A provides the characterization of the search equilibrium, showing how it converges to a competitive equilibrium.

6.2 The Allocation Mechanism in the Search Equilibrium

The allocation mechanism implicit in the search equilibrium (μ_s, M_s, g_s) is constructed as follows:

In a steady-state equilibrium, there is a constant distribution of types in the market. Therefore, the distribution of types leaving the market is the same as the distribution of types entering the market. As all meetings result in trade,[16] these distributions are given by (F, G) with the cutoffs (R_b, R_s).

As prices and allocations depend on who one matches with, the sets of types now include all pairs of buyer types i_b and sellers types i_s, (i_b, i_s). Let y be a profile of net-trades for each possible pair of types of buyers and sellers (i_b, i_s). Let $y(i_b, i_s)$ be the net trade for buyer of type i_b that matches sellers of type i_s and $y(i_s, i_b)$ is the net trade of a seller of type i_s that matched with buyer of type i_b. Let Y be the set of feasible net trades. Then $y \in Y$ is a feasible

[15] For steady-state equilibrium to exist we need to impose some conditions on the distribution of buyer and seller types, for example assuming that $s = b$ and that $N_b = Ns$ are sufficient conditions so that we can find pairs of marginal types that equate supply with demand.

[16] As shown in the Appendix section A, there exists a $\hat{r} > 0$ such that for a discount rate $r \leq \hat{r}$ all meetings result in trade. For simplicity, we focus on steady-state equilibrium with $r \leq \hat{r}$.

profile of net-trades if and only if $y_{(i,j)} + w_i \in X, y_{(j,i)} + w_j \in X$ and $\sum_{(i,j)} y_{(i,j)} = (0,0)$.

Note that as a steady-state equilibrium might feature $R_b < \min \{x(i_b)\}_{i_b=1}^{N_b}$ and $R_s > \max \{z(i_s)\}_{i_s=N_b+1}^{N_s}$; all types of buyers and sellers participate in the market message space of this allocation mechanism must specify a price for each possible pairing of buyers and seller types, which means that there are $N_b \times N_s$ prices for each pair of buyer-seller types (i_b, i_s).

The message space is:

$$M_s = \{(p_s, y) \in \mathcal{R}_+^{N_b} \times \mathcal{R}_+^{N_s} \times Y : p_s(i_b, i_s) y_1(i_b, i_s) + y_2(i_b, i_s) = 0 \;\; \forall (i,j)\}, \tag{16}$$

where $p_s(i_b, i_s)$ is a price assigned for a pair of buyer and seller types. We construct a message correspondence that is privacy preserving and implements the allocation of the steady-state search equilibrium.

Let μ_s^i be a correspondence from the set of environments E^i (here constrained to buyers and sellers with unit demand) to M_s that satisfies $\mu_s^i(e^i) = \{(p_s, y) \in M_s : y(i) = y_s(i)\}$, where $p_s(i_b, i_s)$ is a price for a pair of buyer and seller types (i_b, i_s) and $y_s(i)$ is the net-trade in the steady-state search equilibrium. If i is a buyer of type i_b who meets seller i_s and wants to trade, which means $v_{i_b} > p_s(i_b, i_s)$, the net trade for type i is given by $y_s(i) = (1, -p_s(i_b, i_s))$. Otherwise, if $v_{i_b} < p_s(i_b, i_s)$ then i does not enter the market, trade does no occur, and $y_s(i) = (0,0)$.

Define the correspondence $\mu_s : E \rightrightarrows M_s$ by

$$\mu_s(e) = \cap_i \mu^i(e^i) \cap (p_s(e) \times Y), \tag{17}$$

where $p_s(e)$ is the profile of prices determined by the steady-state search equilibrium in the environment e for the types that trade in equilibrium (so $p_s(e, (i_b, i_s))$ is the equilibrium price for a pair of buyer and seller types (i_b, i_s)), for the types that do not trade in equilibrium set $p_s(e, (i, i_s)) = R_b$ if i is a type of buyer who does not participate in the market for any seller type i_s (note that $v_i < R_b$ so this type does not trade), and $p_s(e, (i_b, i)) = R_b$ if i is a type of seller who does not participate in the market (note that $v_i > R_s$ so this type does not trade as well).

Note that since buyers and sellers meet randomly and the transaction price depends on the pair of valuations of buyers and sellers ($p(x, z)$), prices are not deterministic in the search equilibrium. However, the distribution of realized transaction prices is deterministic, as there is a continuum of consumers. Thus any search equilibrium p_s implies an equilibrium c.d.f. of prices P. Also, note that $p_s(i) > R_s$ if $c^i \leq R_s$ and $p_s(i) < R_b$ if $v^i \geq R_b$ since prices must compensate for search costs, while consumers who do not trade are the types with costs/valuations in (R_s, R_b).

Finally, let the outcome function g_s satisfy $g_s(p, y) = y$, it is a projection from M_s to Y.

6.3 Size of the Message Space

The profile of prices and net trades is a higher dimensional object than under the competitive mechanism. To see this, consider an economy with an even number of N types, with $N/2$ types of potential buyers and $N/2$ types of potential sellers. For the matching and bargaining mechanism in this economy, we need to specify a different price for each pair of types ($i_{b,s}$), so there are $(N/2)^2$ prices for the indivisible good. For the quantities traded, once we have specified the quantity bought by a buyer of type i_b from a seller of type i_s, we have also defined the quantity sold by i_s to i_b. The market clearing condition applies for each pair of trades in the matching environment. Therefore, the message space is $(N/2)^2$ for quantities traded. Therefore, the matching mechanism message space M_s is $2(N/2)^2$ dimensional as stated in Lemma 2 below:

Lemma 2. *The matching mechanism message space M_s^N is a $2(N/2)^2$ dimensional.*

Proof. See Appendix Subsection B.5 □

Combining Lemma 1, which shows that the competitive mechanism message space is N dimensional, and 2, which shows that the matching mechanism message space is $2(N/2)^2$ dimensional, we can see that difference between the matching and competitive mechanisms diverges to infinity as N increases. This is stated as Proposition 7.

Proposition 7. *As $N \to \infty$ the ratio of the dimensional size of M_s^N to M_c^N diverges to infinity.*

(a) Prices

(b) Quantities Traded

$$m = (p(1,1), p(1,2), p(2,1), p(2,2), y(1,1), y(1,2), y(2,1), p(2,2))$$

(c) Message

Figure 3: Example Matching Mechanism with $N = 4$ types of consumers categorized as buyers and sellers.

While Mortensen and R. Wright (2002) show that as the frictions of trade decrease, the distribution of prices across transitions converges to the competitive price, so the matching and bargaining allocation mechanism converges to the competitive mechanism in terms of allocation. However, it does not approximate it in terms of complexity. In other words, the matching and bargaining mechanism requires that each market participant be aware of all types of participants operating in the market to form expectations regarding payoffs from participating in the market and bargaining with the other participants. This is precisely the inverse of the intuition regarding the minimum informational complexity of the competitive market: that each market participant can use prices to substitute for the information regarding other agents they would otherwise require to allocate resources.

To visualize why the message space is larger for a random matching and bargaining mechanism than the competitive mechanism, consider our previous example with four types of consumers, partitioned into two types of buyers and two types of sellers. For the random matching and bargaining mechanism, the oracle now needs to communicate a price for each pair of possible trades, as shown in Figure 3a. She also needs to communicate the quantity traded for each pair, as shown in Figure 3b. Combined, the message space for the search mechanism is 8-dimensional: four prices (one price for each possible pair of buyers and sellers) and four quantities (the quantity sold by the seller to the buyer for each possible pair).

7 Concluding Remarks

We argued that an important factor to consider when evaluating allocation mechanisms is the degree of informational complexity. For example, one justification for using competitive models is that people need such little information to implement a competitive equilibrium.[17]

The size of the messages needed to implement an allocation is an elegant measure of the informational burden placed on the agents in the model. Economists have proved that the competitive allocation mechanism is the only allocation mechanism that minimizes informational complexity. However, as the model of perfect competition assumes that prices are not set by rational agents but are determined as the prices that "equate supply with demand," models of strategic price formation mechanisms are needed to provide strategic foundations for the concept of competitive equilibrium. Thus, we studied informational complexity in allocation mechanisms where the terms of trade are set by strategic agents. We studied two such mechanisms: an allocation mechanism with intermediation (market-maker model) and an allocation mechanism without intermediation (a random matching and bargaining mechanism).

Models of random matching and bargaining have been extensively studied as explanations for how competitive equilibrium allocations might be approximated as frictions of trade decrease (Gale 2000,Mortensen and R. Wright 2002). However, we showed that a random matching mechanism is extremely complex in terms of information: every agent must be able to search across all the people in the economy to find trading partners, so each individual agent must have a complete model of the economy. Thus, it fails to provide strategic foundations for the minimal informational complexity that characterizes the competitive equilibrium.

In contrast, we proposed a strategic allocation mechanism where market-makers intermediate trade. The market-maker mechanism requires almost as little information as the competitive allocation, even when it is used to explain deviations from the competitive allocation. This

[17] The model also needs to fit the data, which, for the competitive model, is most clearly seen from experimental data Smith (1982), Friedman (1984), Friedman and Ostroy (1995), Shachat and Zhang (2017), Martinelli, Wang, and Zheng (2023), and Al–Ubaydli, Boettke, and Brian C Albrecht (2022).

low informational complexity is one possible reason we observe intermediaries facilitating trade between individual sellers and individual buyers: if traders only need to be aware of a few intermediaries for each good they purchase, the informational requirements are much smaller than if traders need to form a model of the whole market before engaging in search and bargaining for their consumption bundle.[18]

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[18] This model allows us to understand better the informational complexity of intermediating "platforms" which are just another term for market-makers Spulber (2019), and their growing role within the modern economy. Our results suggest that the presence of platforms with large market shares, such as Amazon, Google AdSense, and Uber, reduces informational complexity compared to industries with many agents on both sides of the market.

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A Appendix: Characterization of Equilibrium in the Random Matching Economy

The equilibrium of the random matching and bargaining model approximates the frictionless limit when search costs converge to zero (where the Law of One Price holds). To see this, notice that as the discount rate r goes to zero, the left-hand side of 14 does not depend on the buyer's valuation x. Thus, the variation of the left-hand side with regard to x converges to zero as $r \to 0$, which implies that $x - V_b(x)$ converges to a constant as $r \to 0$. In particular, for the marginal buyer type R_b we know that $V(R_b) = 0$; thus, this constant is R_b. Therefore, $x - V_b(x) = R_b$ for $x \geq R_b$ when $r = 0$. Analogously for the seller case, $y + V_s(z) = R_s$ for $y \leq R_s$, which from equation 11 implies that $p(x, z)$ is constant on x and y.

Thus, the Law of One Price holds when the discount rate is zero. That is, if consumers do not discount future payoffs, their expected value in participating in the market is the expected surplus from the future transaction, which varies by the same amount as their valuation. An increase in $\epsilon > 0$ in a buyer's valuation implies an increase in ϵ in their valuation from participating in the market, so $x - V_b(x)$ is constant, and prices are constant in regard to buyers and sellers valuations.

In the frictionless case, as the joint surplus from a meeting is constant

$$[x - z - V_b(x) - V_s(z)] = R_b - R_s. \tag{18}$$

Substituting this expression in equations 14 and 15 when $r = 0$ yields

$$c_b = \frac{m(\theta)(1-\omega)}{\theta} \max\{R_b - R_s, 0\} \tag{19}$$

$$c_s = m(\theta)\omega \max\{R_b - R_s, 0\}, \tag{20}$$

and so $R_b > R_s$. This means that among consumers in the market, any buyer's valuation is higher than any seller's valuation. Therefore, all meetings result in trade since there is no point in searching for other trade opportunities since they are all executed at the same price. In this case, the Law of One Price holds, and substituting equation 18 and $y + V_s(z) = R_s$ into equation 11 we find the equilibrium price:

$$\hat{p} = \omega R_b + (1-\omega)R_s. \tag{21}$$

If search costs (c_b, c_s) converge to zero, then $\hat{p}$ converges to the competitive price and R_s, R_b both converge to the same value R, which is the competitive equilibrium price p^*. In terms of quantity traded, the search equilibrium allocation also converges to $sG(p^*)$, which is the quantity sold in competitive equilibrium.

As equations 14 and 15 are continuous on r, if search costs c_b, c_s are strictly positive and r is lower than some threshold $\hat{r} > 0$, then all meetings result in trade. This implies that the steady-state equilibrium distribution of operating types (Φ, Γ) is given by the densities of (F, G) on the types who participate $(v \geq R_b, c \leq R_s)$. For $r \in (0, \hat{r})$, all meetings result in trade and there is price dispersion as $p(x, z)$ varies with x and y.

To solve for the equilibrium in this case, note that since all meetings result in trade, which implies $\max\{x - z - V_b(x) - V_s(z), 0\} = x - z - V_b(x) - V_s(z)$. If we substitute in equation 14 and differentiate with respect to x, we have that $V_b'(x) = (1-\omega)m(\theta)/[r\theta + (1-\omega)]$, which is constant. Therefore, $V_b(x)$ is linear and setting $V_b'(R_b) = 0$ yields the intercept. Using an analogous procedure, we can solve for $V_s(z)$. Therefore, we have

$$V_b = \frac{(1-\omega)m(\theta)}{(1-\omega)m(\theta)+r\theta}(x - R_b), \tag{22}$$

$$V_s = \frac{\omega m(\theta)}{\omega m(\theta)+r}(R_s - z). \tag{23}$$

Substituting into equation 11 yields the equilibrium price for the transaction:

$$p(x,z) = \omega\left[\frac{r\theta x + (1-\omega)m(\theta)R_b}{r\theta + (1-\omega)m(\theta)}\right] + (1-\omega)\left[\frac{ry + \omega m(\theta)R_s}{\omega m(\theta)+r}\right]. \tag{24}$$

To solve for the set of equilibria where all meetings result in trade, we need to determine $(\hat{r}, R_s, R_b)$. The market-clearing condition that R_s and R_b must satisfy is

$$G(R_s) = [1 - F(R_b)]. \tag{25}$$

Finally, to find R_s and R_b, substitute equation 22 into 14, which gives

$$\frac{c_b\theta}{m(\theta)} = (1-\omega)\int\left[R_b - \frac{ry - \omega m(\theta)R_s}{r+\omega m(\theta)}\right]d\Gamma(z). \tag{26}$$

In the steady-state when all meetings result in trade, the distributions of participating types are $\Gamma(z) = G(z)/G(R_s)$ and $\Phi(z) = F(x)/[1 - F(R_b)]$, and so

$$\frac{c_b\theta}{m(\theta)} = (1-\omega)\int\left[R_b - \frac{ry - \omega m(\theta)R_s}{r+\omega m(\theta)}\right]\frac{dG(z)}{G(R_s)}. \tag{27}$$

Similarly, substituting equation 23 into 13 yields

$$\frac{c_s}{m(\theta)} = \omega\int\left[-R_s - \frac{r\theta x + (1-\omega)m(\theta)R_s}{r\theta + (1-\omega)m(\theta)}\right]\frac{dF(z)}{F(R_s)}. \tag{28}$$

These two equations combined with the market-clearing condition determines (θ, R_s, R_b). Mortensen and R. Wright (2002) show that there is a unique $\hat{r}$ such that every meeting results in trade if and only if $r < \hat{r}$. The condition that every meeting results in trade keeps the model easily tractable with positive search costs and price dispersion in equilibrium.

As shown in Mortensen and R. Wright (2002), as the discount rate r and the search costs (c_b, c_s) both converge to zero, then the search equilibrium prices all converge to p^* and that the search equilibrium converges to the competitive equilibrium. Then, as it is the same allocation mechanism as the competitive equilibrium (as all trades occur at the competitive price), it is informationally efficient at this frictionless limit. We are interested in the allocations implemented by this mechanism away from the limit.

B Appendix: Proofs

B.1 Proof of Lemma 1

Proof. Using the conditions of market-clearing, $\sum_{i=1}^{k} y_i = \mathbf{0}$ and budged balance, $py_1 = 0, \forall i$, implies that the function $(p, y) \to (p, \tilde{y}) \in \mathbb{R}_{++} \times \mathbb{R}_{++}^{N-1}$, where for $1 \leq i \leq N-1$, $\tilde{y}_{i1} = y_i$, is a C^∞-diffeomorphism. Thus, M_c^k is a $(N-1)+1 = N$-dimensional manifold. $\square$

B.2 Proof of Proposition 1

Proof. Posting the competitive price is a Nash equilibrium as it yields zero profits, and a deviation either gives negative profits (if purchase prices are higher than p^* and for selling are lower than p^*) or zero profits (in the case the purchase prices are lower than p^* and for selling are higher than p^*). To see that it is the unique Nash equilibrium, suppose for a contradiction it is not. There exists another Nash equilibrium where market-makers post prices to make strictly positive profits. Other market-makers could deviate and make profits by capturing the customers of competitor market-maker by posting more attractive bid and ask prices. $\square$

B.3 Proof of Proposition 2

Proof. **Part 1. Existence and characterization:**

As accessibility is independent, the competitive equilibrium price p^* is also the competitive equilibrium price for the subset of traders who have access to a market-maker. To construct the candidate equilibrium strategy profile $\{P^j\}_{j \in J}$, we consider pricing strategies described by a pair (p_b, p_s) of offers to buy and sell the good by the market-maker where $p_b \leq p^* \leq p_s$.

First, consider the monopoly prices $\boldsymbol{p}^M = (p_b^M, p_s^M)$ which satisfies the monopolist market-maker problem (which is to maximize profits given by equation 5 subject to the constraint described in equation 6). If there are multiple profit-maximizing pairs of monopoly prices, let (p_b^M, p_s^M) be the pair of monopoly prices with the lowest difference between the buying and selling price that clears the market.

Let $\bar{j}$ be the market-maker with the largest accessibility parameter: $(m^{\bar{j}} = \max\{m^j\}_{j \in J})$. Let

$$\underline{\alpha} = \prod_{h \neq \bar{j}} (1 - m^h),$$

and let Π^M be the monopoly profit (that is, the profits of a market-maker posting the monopoly prices in a situation of monopoly). Consider a function $\boldsymbol{p} : [0,1] \to \mathbb{R}_+^2$ such that $\boldsymbol{p}(\alpha) =$

$(p_b(\alpha), p_s(\alpha))$ is a pair of prices that satisfies

$$\pi(p_b(\alpha), p_s(\alpha)) = \alpha \Pi^M, \tag{29}$$

and satisfies market clearing constraint (described in equation 6).

That is, $(p_b(\alpha), p_s(\alpha))$ is the pair of prices that implements a feasible net trade for a monopolist market-maker and yields a fraction α of the monopoly profits. In addition, if for some $\alpha \in [0,1]$ there is more than one such pair of prices, then $(p_b(\alpha), p_s(\alpha))$ is the pair with the smallest difference between the buying and selling prices.

This is stated formally as follows: for each $\alpha \in [0,1]$, the prices $(p_b(\alpha), p_s(\alpha))$ satisfy

$$(p_b(\alpha), p_s(\alpha)) = \arg\min_{(b,s)} \{|p_s - p_b| : (p_s, p_b) \text{ satisfies equations } 6, 29\}.$$

The existence of at least one pair of prices that satisfies equations 6 and 29 follows from the continuity of consumer demand.

Note also that if a pair of prices is feasible for a monopolist market-maker, then such a pair of prices is also feasible for a market-maker competing with other market-makers. To see this, consider two market-makers in competition. Suppose each market-maker $j \in 1, 2$ posts a pair of bid and ask prices $\boldsymbol{p}(\alpha^j)$ for some $\alpha^j \in [0,1]$. If $\alpha^1 < \alpha^2$, 1 is posting lower ask prices and higher bid prices, and consumers who are aware of both market-makers prefer to trade with 1. Since consumer's preferences are independently distributed from consumer accessibility, the quantities supplied and demanded from 2 fall in the same proportion (both supply and demand from market-maker 2 decreases by a fraction of m^1), so market-clearing still holds.

The candidate equilibrium strategy profile $\{P_j\}_{j \in J}$ is a profile of cumulative distribution functions on $[\underline{\alpha}, 1]$; $P_j(\alpha)$ is the probability that buying (selling) prices higher (lower) than $p_b(\alpha)(p_s(\alpha))$ and that satisfies the equal profit condition

$$\prod_{h \neq j} (1 - P_h(\alpha) m^h) \pi(p_s(\alpha), p_b(\alpha)) = \underline{\alpha} \Pi^M, \tag{30}$$

where $pi(p_s, p_b)$ is given by 5. Note that

$$\underbrace{1 - m^h}_{\text{Prob. } h \notin A^i} + \underbrace{[1 - P_h(\alpha)] m^h}_{\text{Prob.} h \in A^i \text{ and } (p_b^h < p_b(\alpha) \text{ or } p_s^h > p_s(\alpha))} = 1 - P_h(\alpha) m^h, \tag{31}$$

is the probability that a consumer chooses to transact with the market-maker j over competitor h and $\frac{\alpha}{s} \Pi^M$ is the profit margin of a market-maker when posting prices at the minimum profitability level (lower bound for sales, upper bound for purchases), which means its selling (buying) prices undercuts (tops) all competitors'. Also note that equation 30 implies that

$P_h(\underline{\alpha}) = 0$ as $[p_s(\underline{\alpha}) - p_b(\underline{\alpha})]G[p_b(\underline{\alpha})] = \underline{\alpha}\Pi^M$.

To check that this is an equilibrium, note that any prices not in the support of equilibrium strategies $\mathcal{P} = \{(p_b(\alpha), p_s(\alpha)) : \alpha \in [\underline{\alpha}, 1]\}$ lead to strictly lower profits: If we consider prices $(p_b(\alpha), p_s(\alpha))$ that satisfy equations 29 and 6 defined for $\alpha < \underline{\alpha}$, profits are strictly lower by construction. For prices $(p_b, p_s) \in [p_b^M, p_b(\underline{\alpha})] \times [p_s(\underline{\alpha}), p_s^M] \cap \mathcal{P}$, they either yield strictly lower profits because they are undercut by prices which would achieve similar profitability in the case the market-maker were a monopolist, or they are not feasible (that is, the market-maker promises to sell more than it purchases).

Given the sharing rule, it is easy to check that profits are constant on the support of $\{P^j\}$ for each j. If there is only one market-maker j with the largest accessibility parameter m^j, then the equal profit condition (equation 30) implies that there is an atom of probability in the mixed strategy of the largest market-maker at the monopoly price, $P^j(\boldsymbol{p}^M)$. The sharing rule implies that consumers always choose to trade with $h \neq j$ if h posts the monopoly prices $\boldsymbol{p}^M$, hence its profits do not fall discontinuously on the support of the equilibrium strategy $[\underline{\alpha}, 1]$ as $\boldsymbol{p}(\alpha) \to \boldsymbol{p}^M$.

Part 2. Uniqueness:

Note that any feasible pricing strategy for the market-maker must be consistent with market clearing. Note that by construction, the pricing strategies on the set of pairs of prices $\{\boldsymbol{p}(a) : a \in [0, 1]\}$ are weakly dominant, as any feasible pricing strategy (p_b, p_s) yields the same profits as a strategy $\boldsymbol{p}(a)$ for some $a \in [0, 1]$, then sellers and buyers prefer the buying and selling prices $\boldsymbol{p}(a)$. Therefore, for any market-maker posting a pair of prices $(p_b, p_s) \notin \{\boldsymbol{p}(a) : a \in [0, 1]\}$ cannot be a best response to a best response. Hence, any candidate for Nash equilibrium consists of distributions over prices in $\{\boldsymbol{p}(a) : a \in [0, 1]\}$.

Because we are restricted in our candidate equilibrium strategies to prices in $\{\boldsymbol{p}(a) : a \in [0, 1]\}$, the proof of uniqueness of equilibrium is a proof of uniqueness over distributions on $[0, 1]$. Consider an equilibrium strategy profile F, undercutting arguments imply that $F = \{F^j\}_{j \in J}$ is non-degenerate and the upper bound of the support for at least a pair of market-makers must include the monopoly price. The union of the supports for the strategies must be convex; otherwise, market-makers could increase profits by posting prices in the complement of the support. Additionally, the supports for the mixed strategies of individual market-makers must be convex; otherwise, the equal-profit condition will be violated. Note that there cannot be atoms at a lower bound of the support of equilibrium price distributions. If there are no atoms at the lower bound of the support of the distribution, the lower bound of the supports for any pair of market-maker must be the same if the interiors of the supports overlap. This implies that equilibrium strategies for all seller types have convex supports.

As any equilibrium in this environment with mixed pricing strategies satisfies the equal profit condition, the discussion in the preceding paragraph implies that Eq. 30 characterizes any equilibrium where the interiors of the supports of the price distributions overlap. This implies this set of equilibria is unique. To finish the proof, it remains to show that for any pair of market-makers, the interior of the support of mixed pricing strategies must overlap.

To see that suppose, without loss of generality, that there is an equilibrium strategy profile $\boldsymbol{P}$ such that there is a pair of market-makers j, j' with the same accessibility parameter $0 < m^j = m^{j'} < m^k$ who compete against each other posting prices according to a strategy that is described by pair of distributions $P^j, P^{j'}$ on $[0, 1]$ and the price posting function $\boldsymbol{p}$ that maps $[0, 1]$ into pairs of buying and selling prices. The distributions $P^j, P^{j'}$ have the same support $[\underline{\alpha}^*, \overline{\alpha}^*]$ while all other market-makers post prices according to distributions that have their supports in $[\underline{\alpha}, 1]$ with $\underline{\alpha} = \overline{\alpha}^*$. In words, market-makers j and j' compete by posting strictly more attractive prices to buyers and sellers than all others. Let $\overline{K}$ be the market-maker with largest accessibility parameter in the subset of market-makers $\hat{J} = J - \{j, j'\}$.

Let $\Pi^o(\alpha)$ be the equilibrium profits per unit of accessibility of market-maker o in posting prices $\boldsymbol{p}(\alpha)$; we call that o's equilibrium "profitability." Since $\overline{\alpha}^* = \underline{\alpha}$, the profitability of market-maker j of posting prices $\boldsymbol{p}(\overline{\alpha}^*)$ is

$$\Pi^j(\overline{\alpha}^*)/m^j = (1 - m^{j'})\overline{\alpha}^*\Pi^M. \tag{32}$$

If market-maker $o \neq j, j'$ posts prices $\boldsymbol{p}(\underline{\alpha})$, its equilibrium profitability is

$$\Pi^o(\underline{\alpha}) = (1 - m^{j'})(1 - m^j)\underline{\alpha}\Pi^M. \tag{33}$$

Note that $\underline{\alpha} = \overline{\alpha}^*$, and therefore the equal-profit condition for j and equation 32 imply that

$$\underline{\alpha}^* = (1 - m^{j'})\underline{\alpha}. \tag{34}$$

Finally, equations 33 and 34 together imply that for market-maker $o \neq j, j'$ that its profitability in posting prices $\boldsymbol{p}(\underline{\alpha}^*)$ satisfies

$$\Pi^o(\underline{\alpha}^*) = \underline{\alpha}^*\Pi^M \tag{35}$$

$$= (1 - m^{j'})\underline{\alpha}\Pi^M > (1 - m^{j'})(1 - m^j)\underline{\alpha}\Pi^M \tag{36}$$

$$= \Pi^o(\underline{\alpha}). \tag{37}$$

The inequality 36 is a contradiction with $\boldsymbol{P}$ being an equilibrium. Therefore, in equilibrium, the interior of the supports must overlap.

Hence, the pricing strategy described by $\{P_j\}_{j\in J}$ is the unique Nash equilibrium given the sharing rule that the market-maker with the smaller accessibility parameter m^j has priority in transactions to buyers and sellers in the case of a tie in prices. $\square$

B.4 Proof of Proposition 5

Proof. As in the proof of proposition 2, let Π^M be the monopoly profit rate. Consider a profit rate $\pi \in (0, \Pi^M)$ (in slight abuse of notation), and suppose the incumbent 1 considers posting prices $\boldsymbol{p}(\pi) = (p_b(\pi), p_s(\pi))$, which is the pair of bid and ask prices with the smallest difference that satisfies $\pi(p_b(\pi), p_s(\pi)) = \pi$. The pricing strategy, $\boldsymbol{p}(\pi)$, yields a payoff of $\pi \times m^j$, if j is a monopolist. We will say a firm "undercuts" by posting a pair of bid and ask prices $\boldsymbol{p}(\hat{\pi})$ with $\hat{\pi} < \pi$, thus $p_b(\hat{\pi}) > p_b(\pi)$ and $p_s(\hat{\pi}) < p_s(\pi)$.

First, for simplicity, we consider the case where agents are myopic and only care about present payoffs. Then, we extend the equilibrium to the case when agents care about future payoffs.

Step 1: One period deterrence game

In this case, agents are myopic and only care about present payoffs so that the discount factor $\beta = \frac{1}{1+r} = 0$. In this case, the deterrence game has only one period. For $\pi \leq E/m_e$, if the incumbent posts $\boldsymbol{p}(\pi)$, then the cost of entry E is higher than the profits 2 can make after entry by undercutting 1 with higher bid and lower ask prices. Therefore, 2 does not enter if 1 posts $\boldsymbol{p}$.

Therefore, if the incumbent posts $\boldsymbol{p}(\pi)$ for $\pi = E/m_e$ (the highest profit margin that deters entry) and the entrant is playing "no entry," this is an equilibrium if 1 has no incentive to deviate. Clearly, 1's profits decrease with a lower bid-ask spread than $\pi = E/m_e$, so there is no incentive for 1 to deviate by posting more attractive prices to the consumers. While prices $\boldsymbol{p}(\pi')$ for $\pi' > \pi = E/m_e$ imply that 2 can make profits higher than E/m_e if 2 enters and undercuts 1. Thus, 1 has no incentive to deviate in pure strategies.

It remains to show that 1 finds it more profitable to deter entry than to compete with 2 in mixed strategies, as described in Proposition 2. Note that the profit that 1 makes in the mixed strategy equilibrium with both market-makers operating is $(1 - m_e)\pi^M$. The profit 1 makes with entry deterrence strategy is E/m_e, thus if the entry cost E is high enough so that

$$E/m_e > (1 - m_e)\pi^M \tag{38}$$

the market-maker 1 finds it profitable to deter entry. Note also, that $m_e(1 - m_e)\pi^M$ are the profits of 2 if they enter and compete in mixed strategies with 1, thus if $E > m_e(1-m_e)\pi^M$, 2 does not want to enter the market even if 1 plays the mixed strategy of the equilibrium where both market-makers compete. If $E \leq m_e(1-m_e)$, 2 finds it a best response to enter the market and compete with 1 if 1 is playing the mixed strategy, and 1's profits in the mixed strategy equilibrium are equal or higher than profit $\pi = E/m_e$ of the deterrence strategy.

Therefore, the deterrence strategy $\boldsymbol{p}(\pi)$ for $\pi = E/m_e$ for 1 and 2 chooses to not enter is the only equilibrium if and only if $E > m_e(1-m_e)\pi^M$.

Step 2: Infinite horizon deterrence game

The one period case can be extended to a dynamic environment. Agents have a common discount factor $\beta \in (0,1)$. Then, payoffs of 1 and 2 playing the mixed pricing strategies in the Markov perfect equilibrium after entry are, respectively

$$U_e^1 = \sum_{t=0}^{\infty} \beta^t (1-m_t^2)\pi^M, \tag{39}$$

$$U_e^2 = \sum_{t=0}^{\infty} \beta^t m_t^2 (1-m_t^2)\pi^M, \tag{40}$$

where t is the number of periods after the entry. Therefore, $\{m_t^2\}_t$ is the sequence of accessibility parameters for 2 that satisfies equation 10 for $t > 0$ and $m_0^2 = m_e$.

Suppose the monopolist can only choose a fixed pricing schedule, posting $\boldsymbol{p}(\pi), \pi \in [0, \pi^M]$ in every period. The present value of 2's profits conditional on entry when 1 is following its commitment $\boldsymbol{p}(\pi)$ is bounded above by

$$U_d^2(\pi) = \sum_{t=0}^{\infty} \beta^t m_t^2 \pi.$$

To deter entry, π must imply that $U_e^2(\pi) \leq E$. Consider the profit-maximizing strategy of entry deterrence π that satisfies $U_e^2(\pi) = E$, substituting for B.4 and rearranging imply that $\pi = E/\left(\sum_{t=0}^{\infty} \beta^t m_t^2\right)$. Thus, payoffs for the deterrence strategy for 1 are

$$U_d^1 = \frac{\pi}{(1-\beta)} = \frac{E}{(1-\beta)\left(\sum_{t=0}^{\infty} \beta^t m_t^2\right)} \tag{41}$$

In equilibrium with entry deterrence the monopolist must find deterring entry profitable: $U_d^1 \geq U_e^1$. Clearly, equation 41 implies that for an entry cost E high enough $U_d^1 > U_e^1$.

By assumption, m_t^2 converges to 1 at a fast enough rate so that

$$\sum_{t=0}^{\infty} (1-m_t^2) \leq C.$$

Now take a sequence $\{\beta_n\}$ such that $\lim \beta_n = 1$. Let $(U_d^1(\beta_n), U_d^2(\beta_n), U_e^1(\beta_n), U_e^2(\beta_n))$ be the corresponding payoffs for 1 and 2 in deterrence and in the equilibrium with entry at the discount rate β_n. Let $\{\pi_n\}_n$, with

$$\pi_n = E/\left(\sum_{t=0}^{\infty} \beta_n^t m_t^2\right) \tag{42}$$

for each n, be the corresponding sequence of candidate deterrence equilibrium profit margins for the monopolist.

Set the entry cost $E \geq C \times \pi^M$. Then profits of 1 and 2 if both enter and play the mixed strategy equilibrium are bounded up by $C\pi M$, and so 2's profits are always lower than the entry costs. Therefore, if 1 sets bid and ask prices $\boldsymbol{\pi_n}$, 2 finds it optimal not to enter. Without entry, equation 41 implies that profits for 1, $U_d^1(\beta_n)$ are greater than E. Thus, if $E \geq C \times \pi^M$, the unique equilibrium is for 1 to deter entry, analogously to the one period case. Note that $\sum_t (1 - m_t^2) \leq C$ and Eq. 42 imply that $\pi \to 0$ as $\beta \to 1$ and therefore $\boldsymbol{p}(\pi)$ converges to $p_s = p_b = p^*$ as the discount rate r falls to zero and the equilibrium allocation must converge to the competitive equilibrium.

Finally, relax the restriction that the monopolist is restricted to posting the same bid and ask prices for every period but chooses a sequence of bid and ask prices. Then, the overall situation is similar: the monopolist chooses a sequence of profit shares $\{\pi_t\}_{t=0}^{\infty}$ with corresponding sequence of pairs of bid and ask prices $\boldsymbol{p}(\pi_t)$. To deter entry the sequence $\{\pi_t\}$ must satisfy

$$\sum \beta^t m_t^2 \pi_t \geq U_e^2, \tag{43}$$

the profits of the monopolist under this strategy are

$$U_d^1 = \sum \beta^t \pi_t. \tag{44}$$

The profit-maximizing strategy for the monopolist is to choose, out of the sequences that satisfy the deterrence condition 43, the one that maximizes equation 44. Given that $m_t^2 \to 1$ and is strictly increasing, there is a unique profit-maximizing sequence $\{\pi_t\}$, where the monopolist "frontloads" by extracting the highest profits in the early periods as the entrant's profits from undercutting are relatively constrained by m_t^2 being smaller than in later periods from taking advantage of these higher margins. These profits are strictly higher than the profits from the strategy to commit to constant prices $(\pi/(1-\beta))$, and therefore the previous arguments also apply in this case. $\square$

B.5 Proof of Lemma 2

Proof. With $N/2$ types of buyers and $N/2$ types of sellers, the price vector of the steady-state search equilibrium has $(N/2)^2$ dimensions (as prices are defined for pairs of buyers and sellers), while Y^s has $(N/2)^2$ dimensions. By analogous argument as for the competitive mechanism in Lemma 1, M_s is a $2(N/2)^2$-dimensional manifold. $\square$